Diffractometry: interpreting diffraction-dominated data

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European Research Council
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KERNEL

Outline

Astronomical images

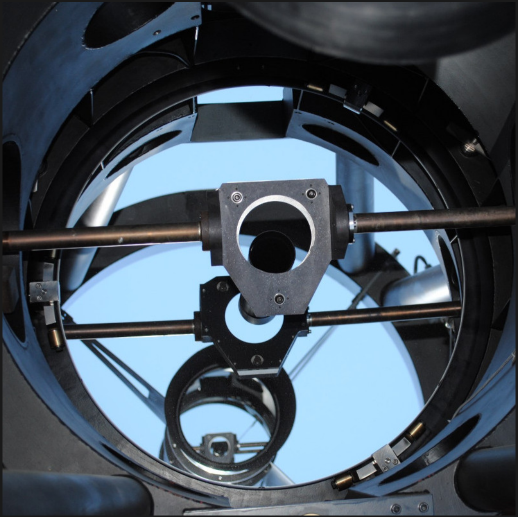


Image formation

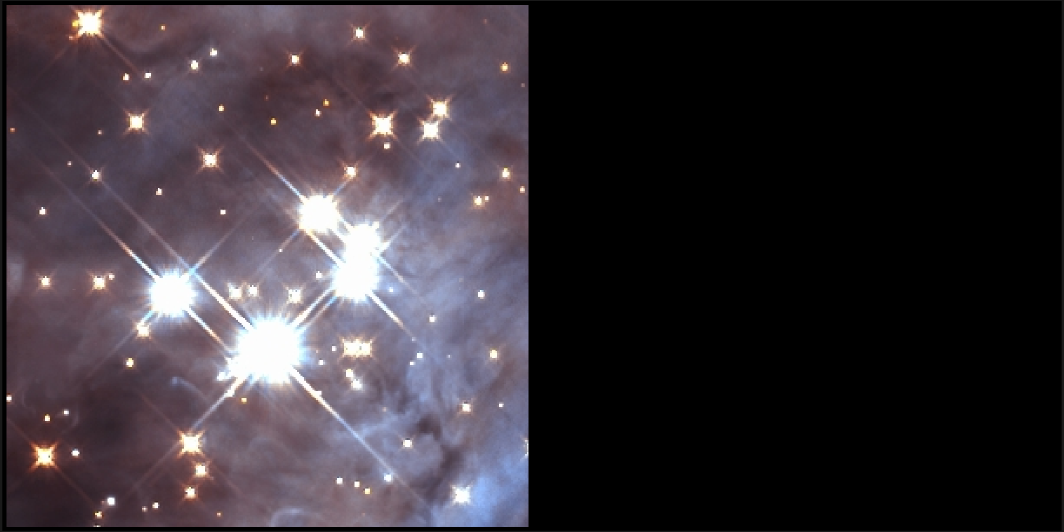


Image formation

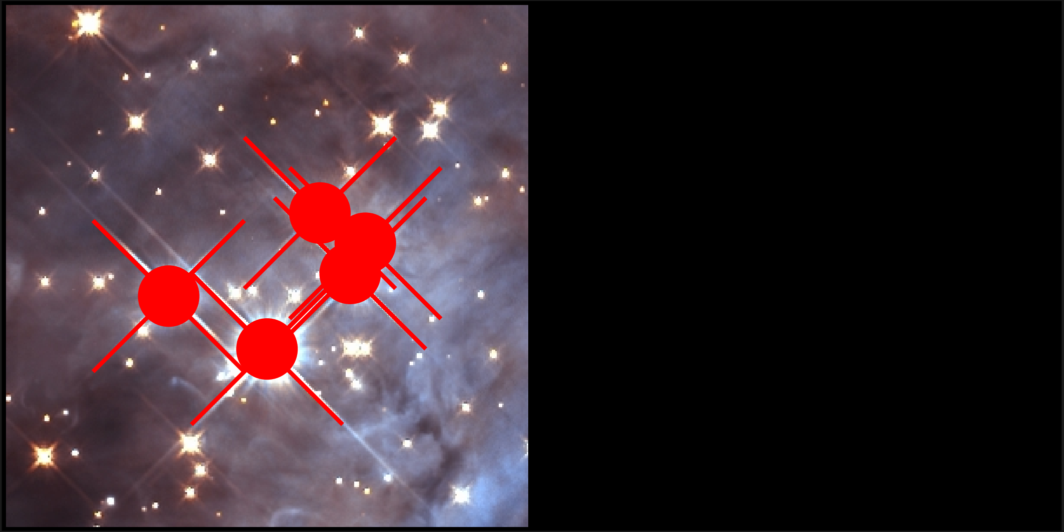


Image formation

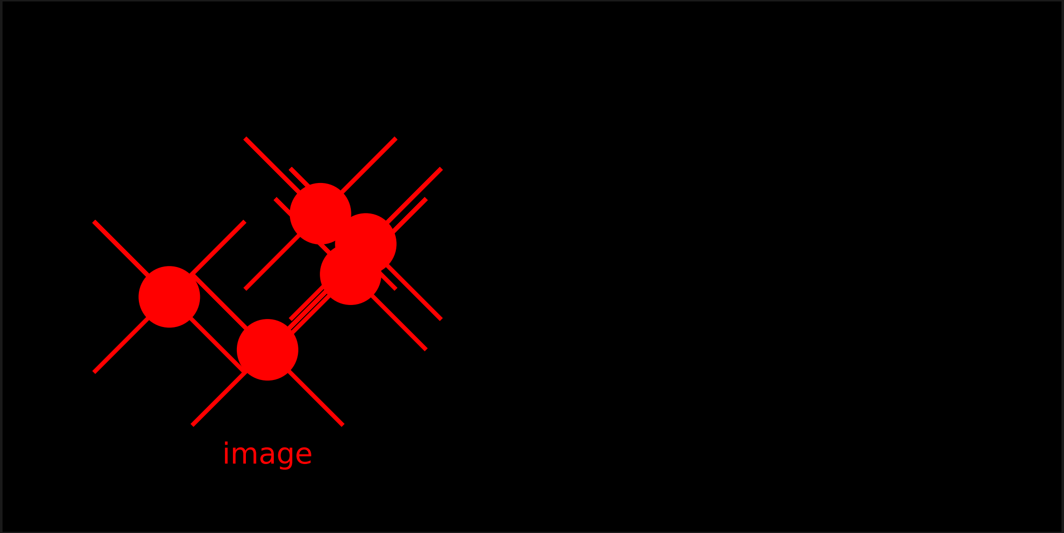


Image formation

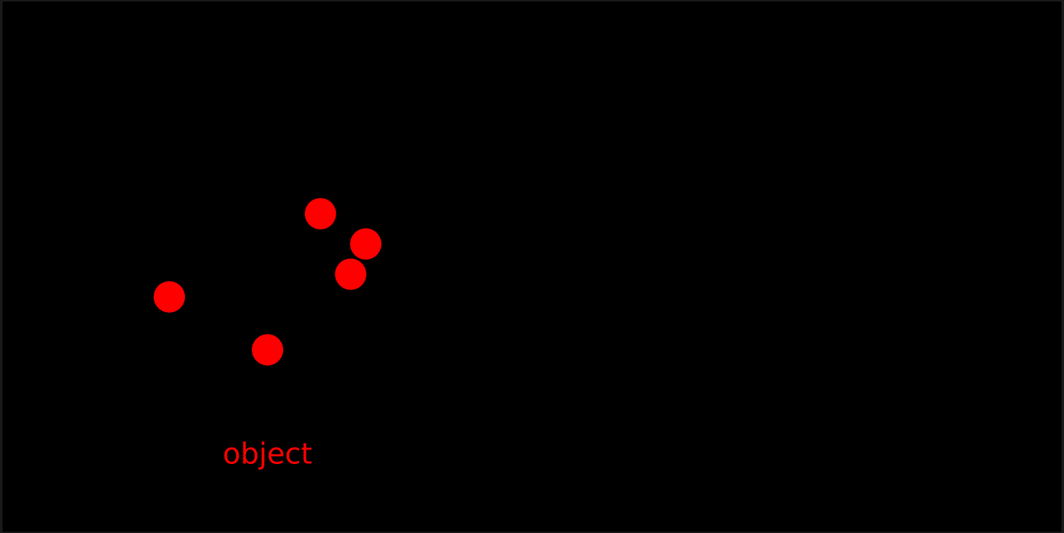
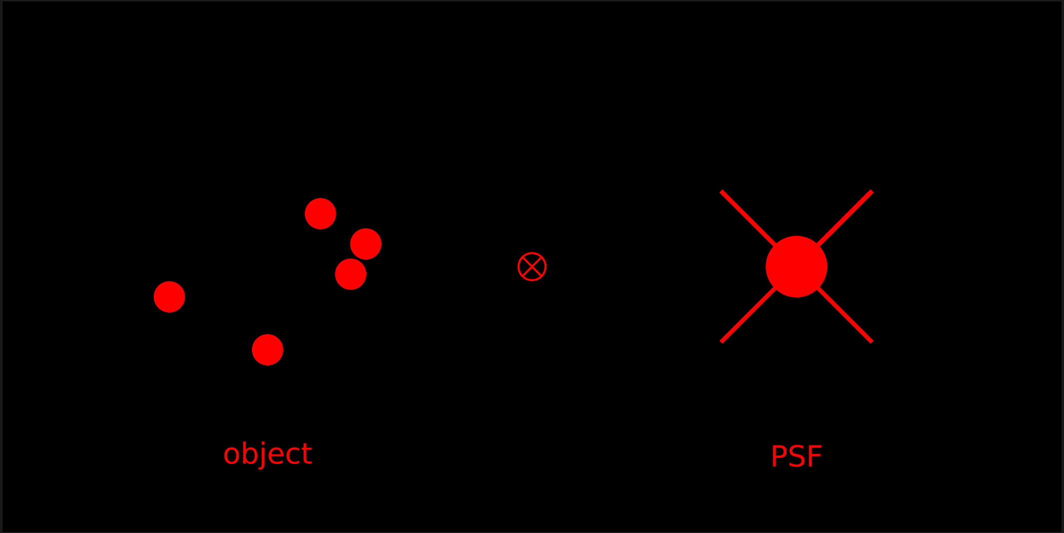
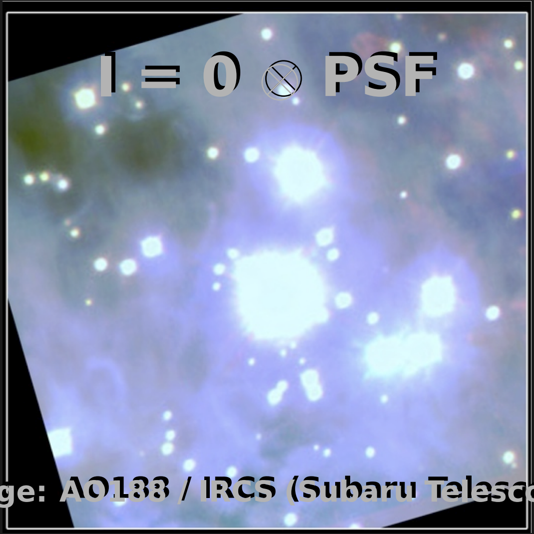


Image formation



Diffraction dominated images



[image: AO188 / IRCS (Subaru Telescope)]

Diffraction dominated images

[image: AO188 / IRCS (Subaru Telescope)]

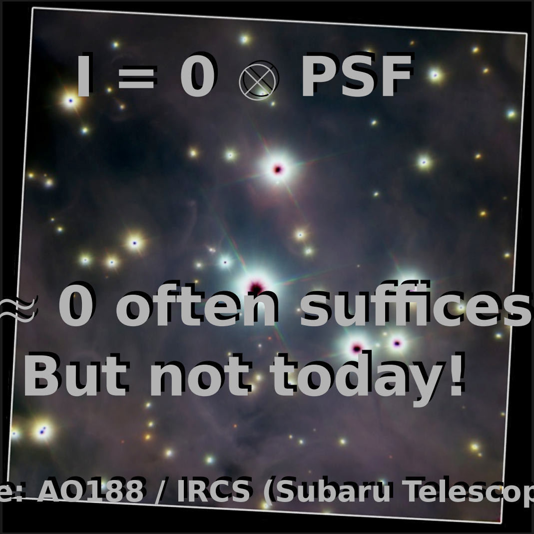
Diffraction dominated images

$$I = O \otimes PSF$$

$$I \approx O \text{ often suffices}$$

[image: AO188 / IRCS (Subaru Telescope)]

Diffraction dominated images



$I = O \otimes PSF$

$I \approx O$ **often suffices**

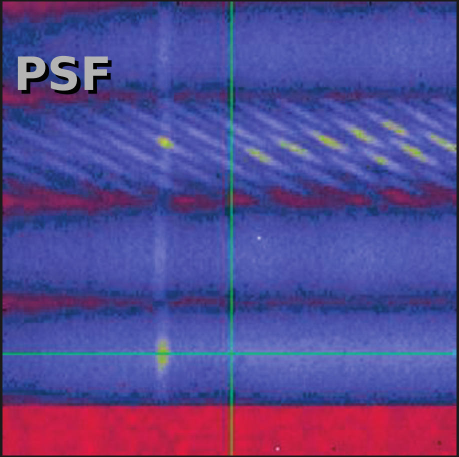
But not today!

[image: AO188 / IRCS (Subaru Telescope)]

Diffraction dominated astronomy

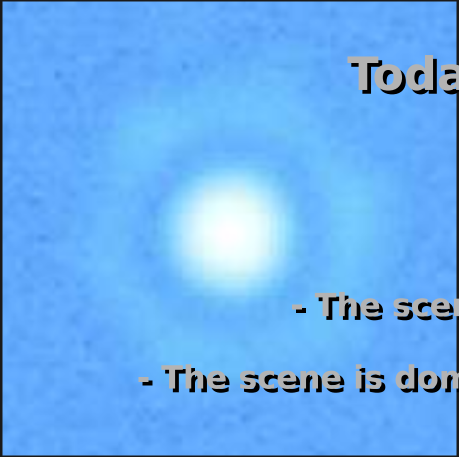
[NIRC2 / Keck]

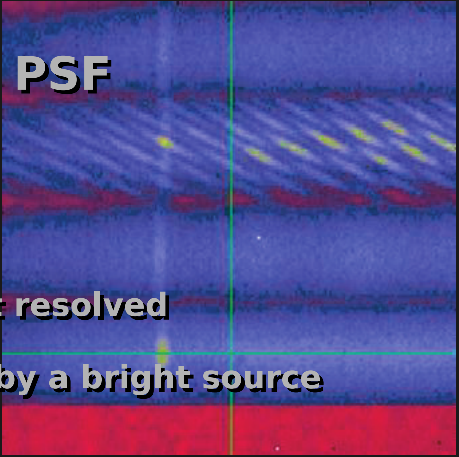
Today: I ≈ PSF



[AMBER / VLTI]

Diffraction dominated astronomy



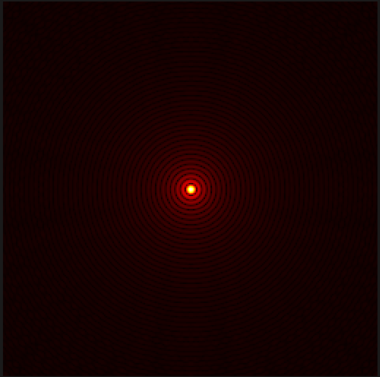


Today: I ≈ PSF

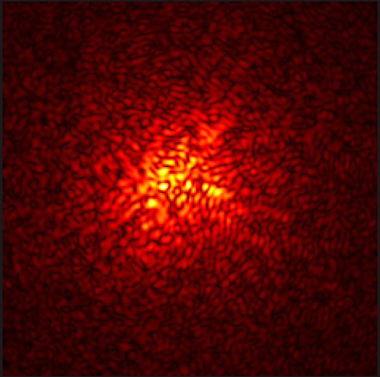
- The scene is not resolved
- The scene is dominated by a bright source

[NIRC2 / Keck]   [AMBER / VLTI]

Observing through the atmosphere



Theoretical diffraction-limited point spread function.



Experienced instantaneous seeing-limited point spread function

Unless something is done about it, a telescope larger than a certain size (typically $\sim$ 0.1 m) produces images limited by the seeing.

An old story

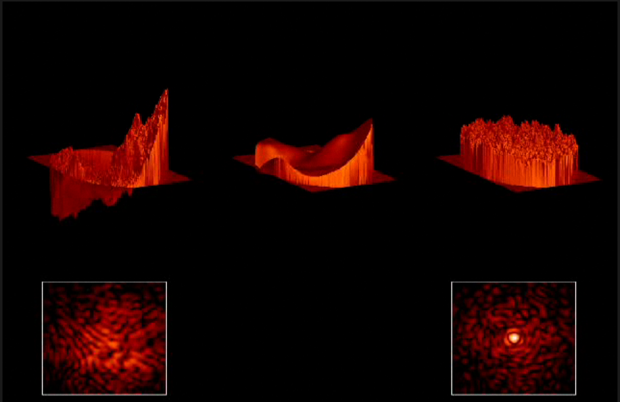
Opticks, Isaac Newton (1704)

"If the Theory of making Telescopes could at length be fully brought into Practice, yet there would be certain Bounds beyond which Telescopes could not perform. For the Air through which we look upon the Stars, is in a perpetual Tremor [. . .]

The only Remedy is a most serene and quiet Air, such as may perhaps be found on the tops of the highest Mountains above the grosser Clouds."

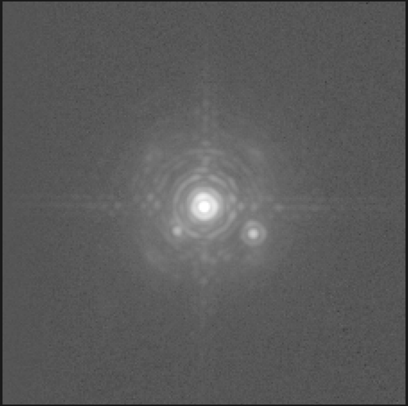
Book I, Prop. VIII, Prob. II

Turbulence filtering with Adaptive Optics



[Animation: James Lloyd]

The Strehl ratio



A very nice AO corrected image

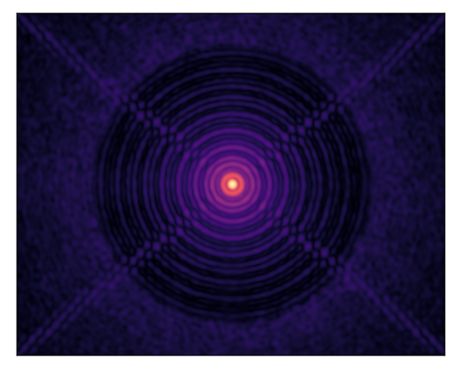
Image quality is often summarized by a single number: the Strehl ratio.
Maréchal approximation:

$$S \approx \exp\left(-(2\pi\sigma/\lambda)^2\right)$$

In the near-IR:

RMS aberration	Strehl
σ = 200 nm	S $\sim$ 0.5
σ = 100 nm	S $\sim$ 0.86
σ = 50 nm	S $\sim$ 0.96

imaging through residual turbulence

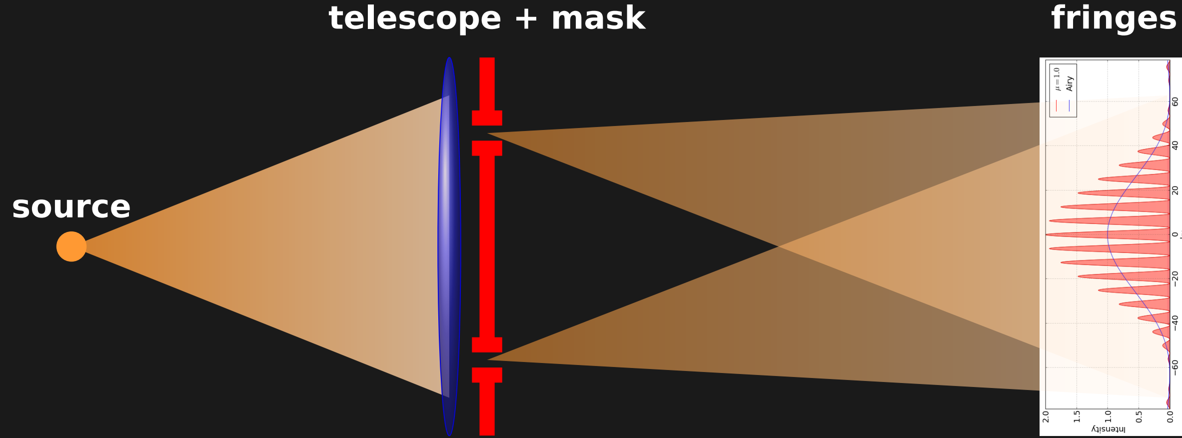


- $I(t) = O \otimes PSF(t)$
- the PSF dominates the signal recorded
- the signal O is a weak perturbation
- small time-varying changes in the PSF dominate

It's a tricky problem → let's simplify it

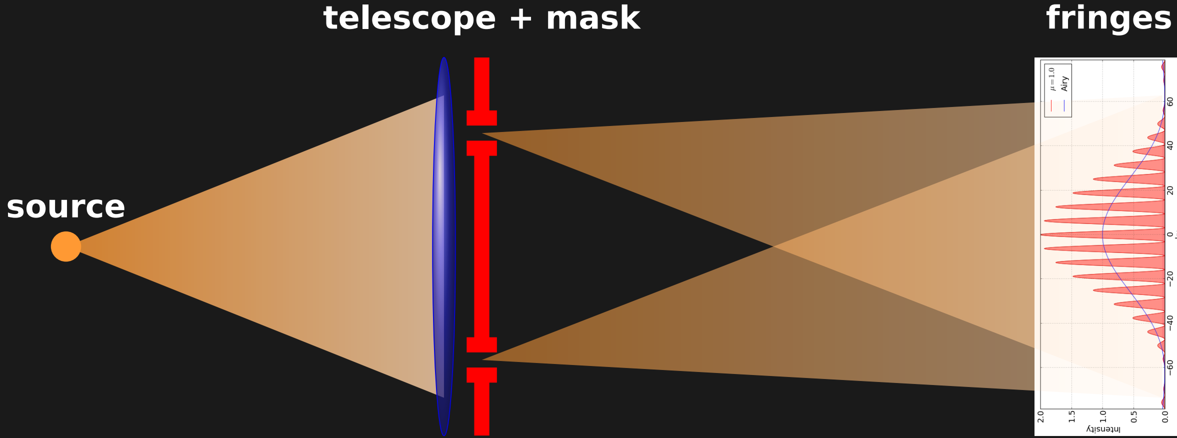
Outline

Simplify the interpretation of images



By simplifying the diffractive aperture!

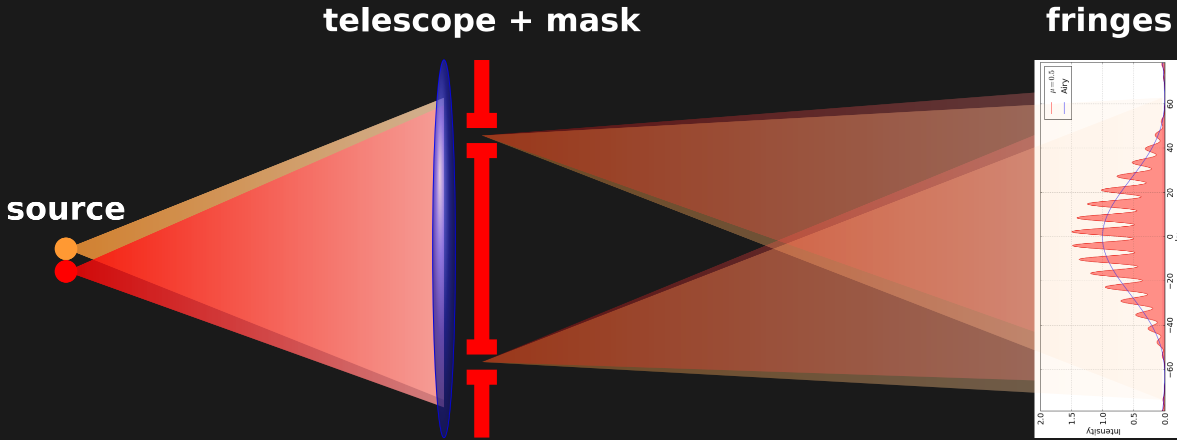
Simplify the interpretation of images



By simplifying the diffractive aperture!

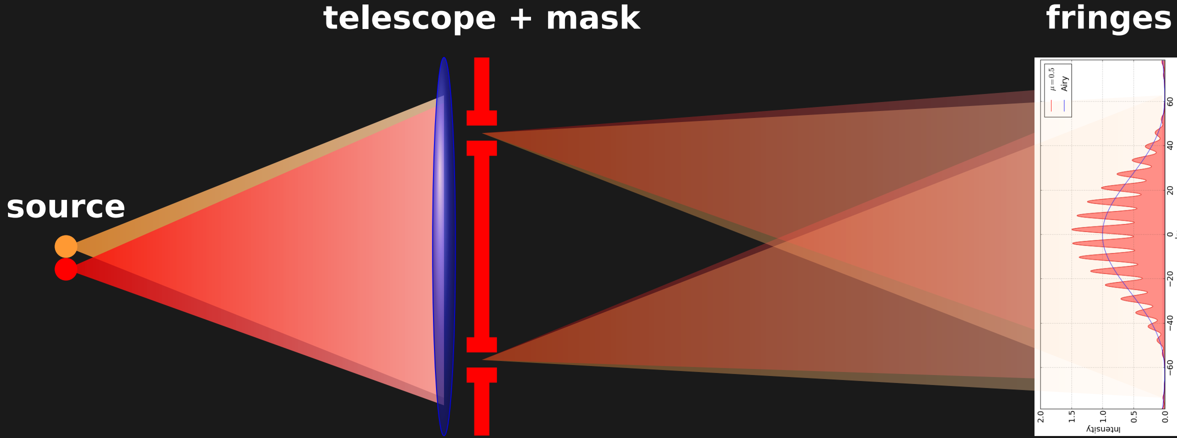
$$I(x) = I_0 \times \left(1 + \mu \cos\left(kx - \phi\right)\right)$$

Interferometric measurements



$$I(x,t) = I_0 \times \left(1 + \mu \cos\left(kx - \phi + \Delta\varphi(t)\right)\right)$$

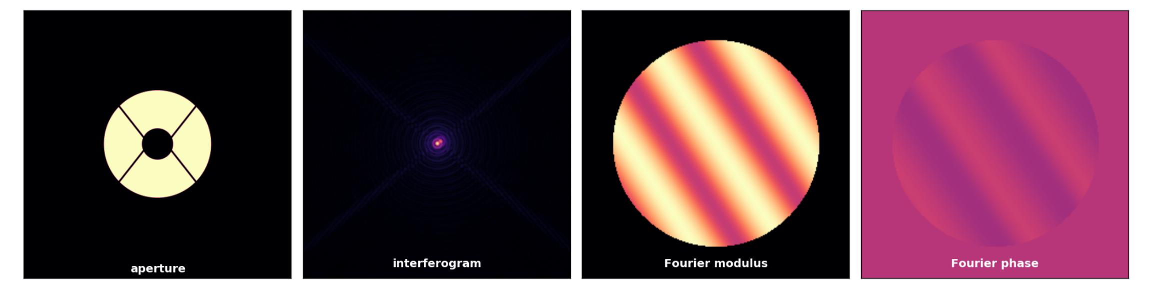
Interferometric measurements



$$I(x,t) = I_0 \times \left(1 + \mu \cos\left(kx - \phi + \Delta\varphi(t)\right)\right)$$

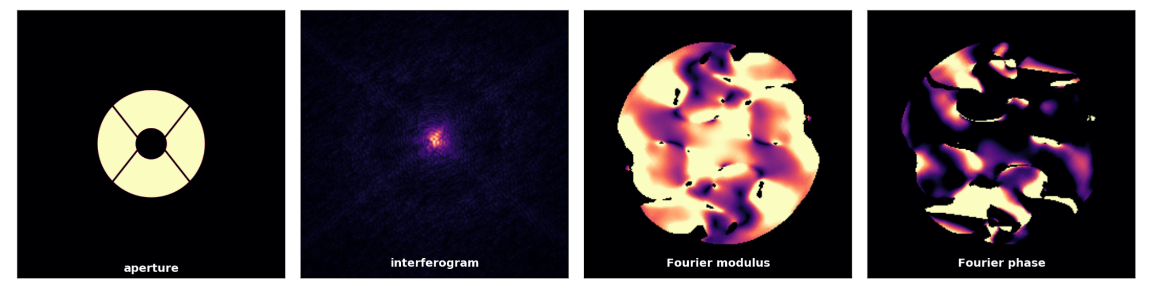
Even if perturbations are still present

The image and its Fourier counterpart



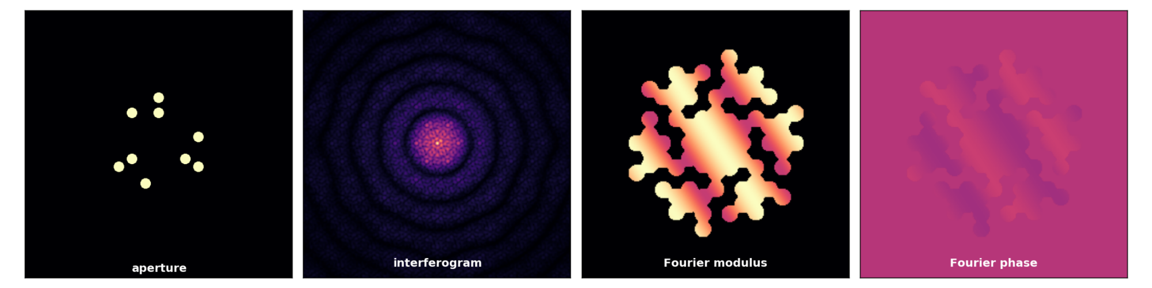
The pupil the image The image Fourier-transform

Perturbed spatial frequencies

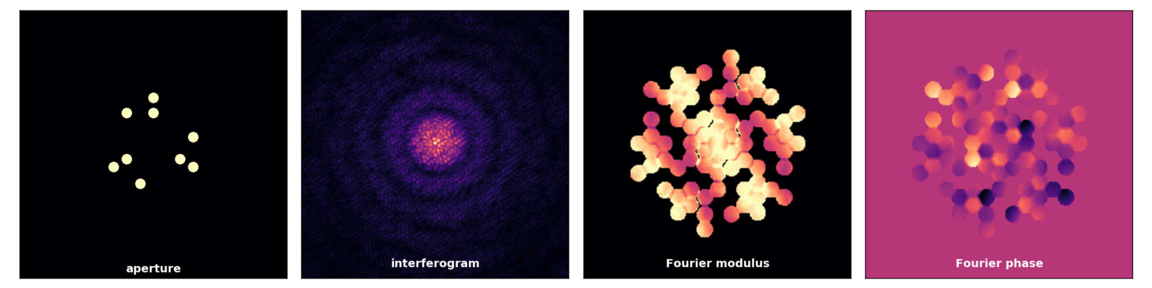


An image and its Fourier transform in the presence of aberrations

Sparse aperture: simplified interpretation

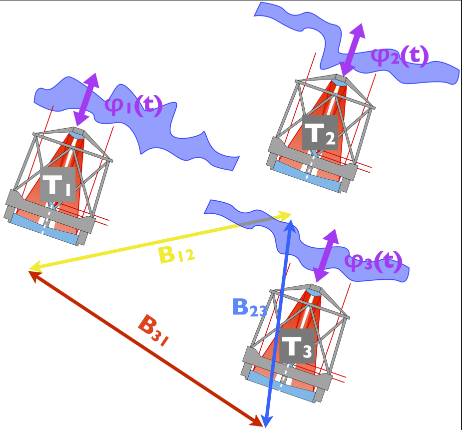


Sparse aperture: simplified interpretation



Even in the presence of turbulence:
Non-redondant aperture $\rightarrow$ visibility modulus
Phase information however still lost

Phase and closure-phase

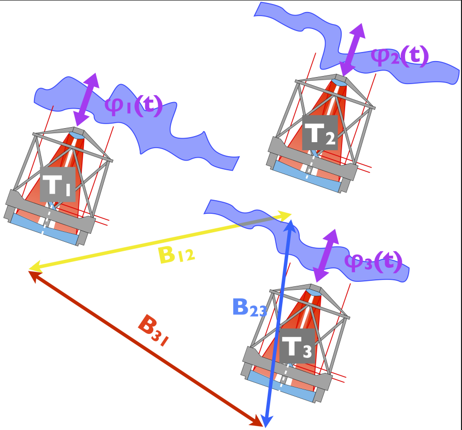


$$\Phi(1-2) = \Phi_O(1-2) + (\varphi_1 - \varphi_2)$$

$$\Phi(2-3) = \Phi_O(2-3) + (\varphi_2 - \varphi_3)$$

$$\Phi(3-1) = \Phi_O(3-1) + (\varphi_3 - \varphi_1)$$

Phase and closure-phase



$$\Phi(1-2) = \Phi_O(1-2) + (\varphi_1 - \varphi_2)$$

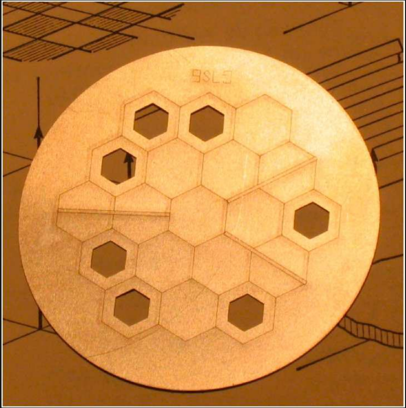
$$\Phi(2-3) = \Phi_O(2-3) + (\varphi_2 - \varphi_3)$$

$$\Phi(3-1) = \Phi_O(3-1) + (\varphi_3 - \varphi_1)$$

The sum of these three terms is independent from the perturbation term: hail the closure-phase!

Jennison, 1958

More complex apertures?



Interferometric mask of JWST
Sivaramakrishnan et al, 2010

- n_A: the number of sub-apertures
- n_B: the number of baselines
- n_C: the number of closure-phases

- $n_B \leq n_A \times (n_A - 1) / 2$
- $n_C \leq (n_A - 1) \times (n_A - 2) / 2$

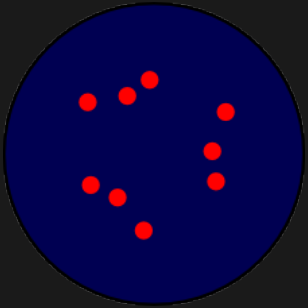
Are all added baselines equal?

See following interactive tools:

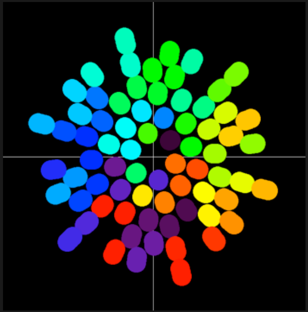
- Interferometric Synthetic PSF
- UV coverage

Outline

Interferometry without a sparse aperture mask?

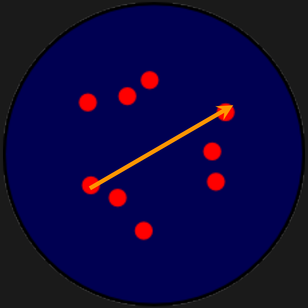


Non-redondant

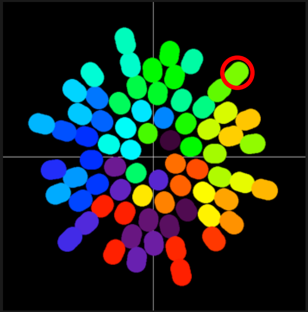


uv coverage

Interferometry without a sparse aperture mask?

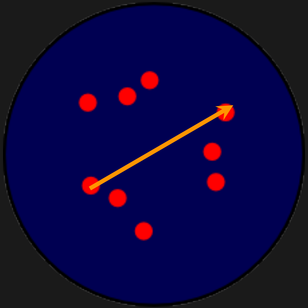


Non-redondant



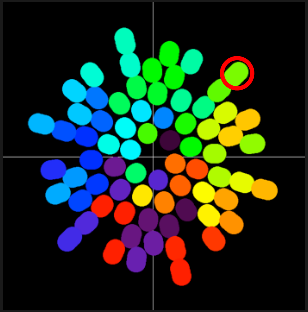
uv coverage

Interferometry without a sparse aperture mask?



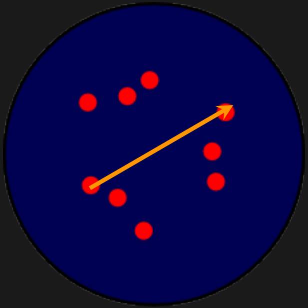
Non-redondant

full aperture



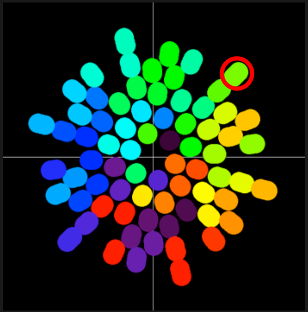
uv coverage

Interferometry without a sparse aperture mask?



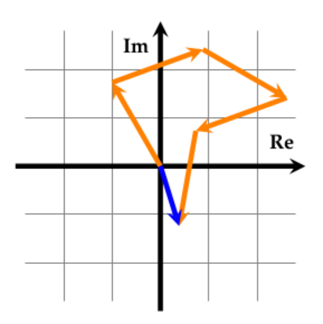
Non-redondant

full aperture



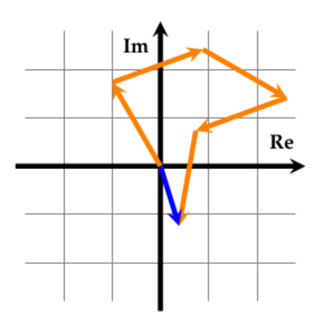
uv coverage

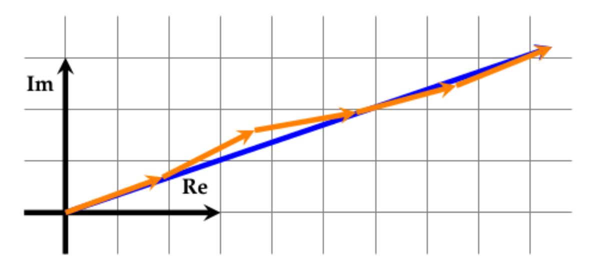
Redundancy



Each phasor contains a strong random component. The measured phase is useless.

Redundancy

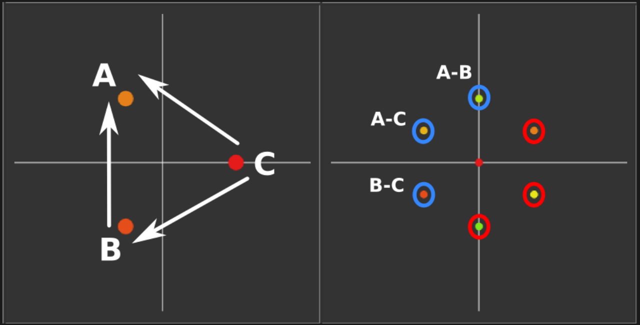




Each phasor contains a strong random component. The measured phase is useless.

With Adaptive Optics, phasors do line up. The phase of the sum of these phasors is a good proxy for the true phase.

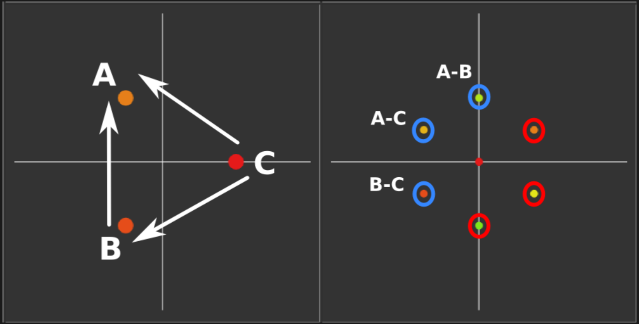
Elementary model 1: triangle



phase relations

- $\Phi(A\text{-}B) = \Phi(A\text{-}B)_0 + (\varphi_A - \varphi_B)$
- $\Phi(A\text{-}C) = \Phi(A\text{-}C)_0 + (\varphi_A - \varphi_C)$
- $\Phi(B\text{-}C) = \Phi(B\text{-}C)_0 + (\varphi_B - \varphi_C)$

Elementary model 1: triangle - revised



linear model

$$\Phi = \Phi_0 + A \cdot \varphi$$

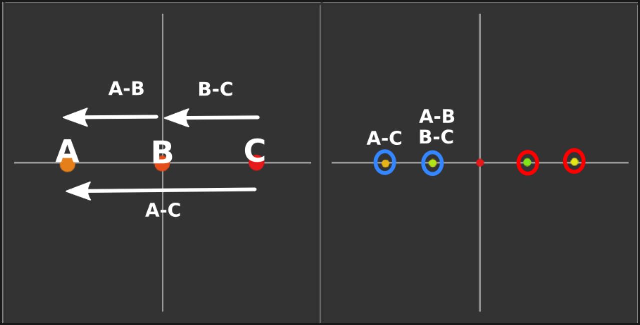
$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

kernel = closure-phase

$$\mathbf{K} = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}$$

Verifies $\mathbf{K} \cdot \mathbf{A} = 0$

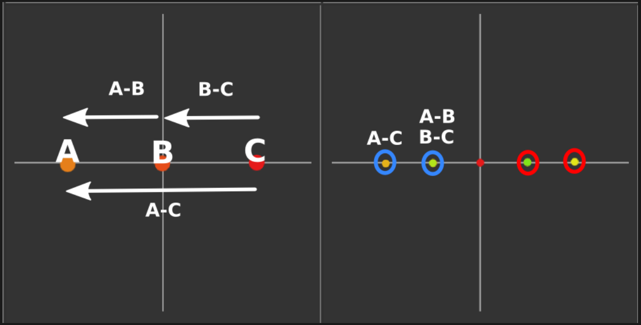
Elementary model 2: in line



phase relations

- $\Phi(A - C) = \Phi(A - C)_0 + (\varphi_A - \varphi_C)$
- $\Phi(B - C) = Arg\left(e^{i(\Phi_0 + (\varphi_A - \varphi_B))} + e^{i(\Phi_0 + (\varphi_B - \varphi_C))}\right)$

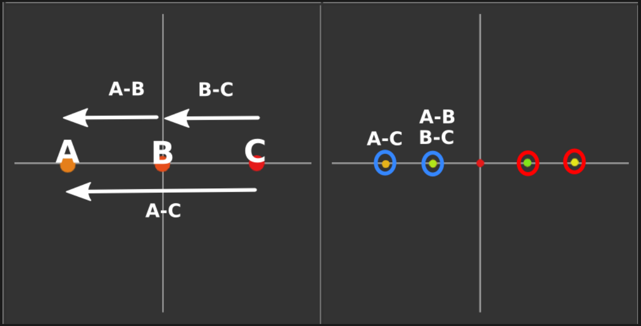
Elementary model 2: in line - linearised



linearized relations

- $\Phi(A-C) = \Phi(A-C)_0 + (\varphi_A - \varphi_C)$
- $\Phi(B-C) \approx \Phi(B-C)_0 + 1/2 \times (\varphi_A - \varphi_C)$

Elementary model 2: in-line - revised



linear model

$$\Phi = \Phi_0 + R^{-1} \cdot A \cdot \varphi$$

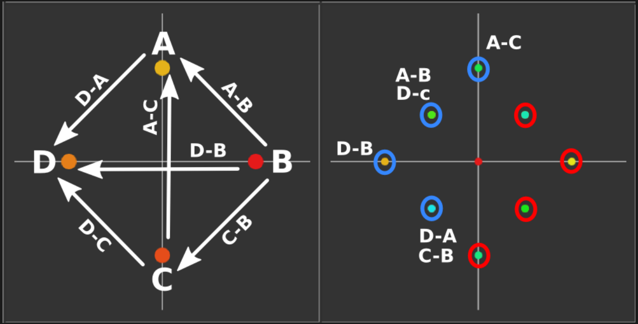
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} R = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

kernel

$$\mathbf{K} = \begin{bmatrix} 1 & -2 \end{bmatrix}$$

Verifies $\mathbf{K} \cdot \mathbf{R}^{-1} \cdot \mathbf{A} = 0$

Elementary model 3: the square



linear model

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

kernels

$$\mathbf{K} = \begin{bmatrix} 1 & -2 & 0 & 1 \\ 1 & 0 & -2 & -1 \end{bmatrix}$$

Revisit the convolution relation

The general Fourier-component phase equation:

$$\Phi^k = \Phi_O^k + Arg\left(\sum_{i=0}^{r} \exp\left(j\Delta\varphi_i^k\right)\right),$$

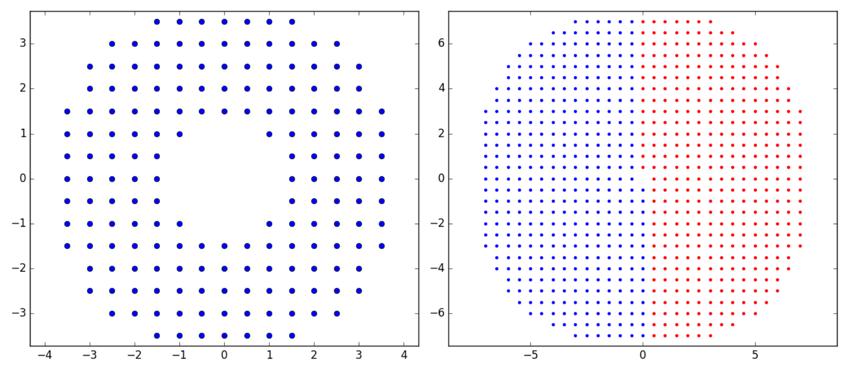
can be linearized in the presence of AO:

$$\Phi^k \approx \Phi_O^k + \frac{1}{r}\sum_{i=0}^{r} \Delta\varphi_i^k$$

The image object convolution relation can be reformulated:

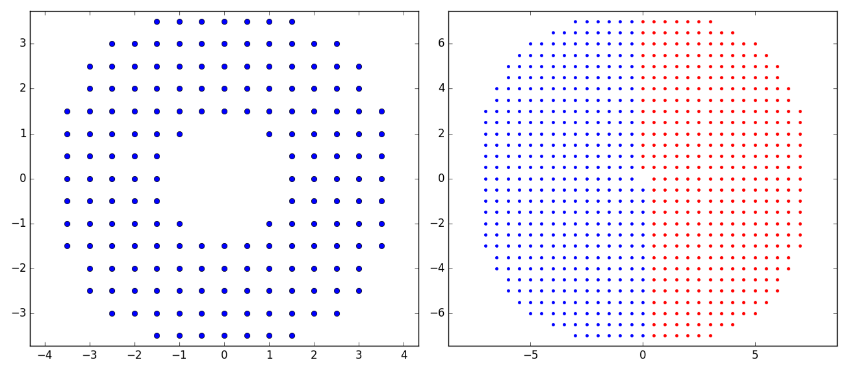
$$I = O \otimes PSF \rightarrow \Phi = \Phi_0 + \mathbf{R}^{-1} \cdot \mathbf{A} \cdot \varphi$$

A different way to look at AO-corrected images



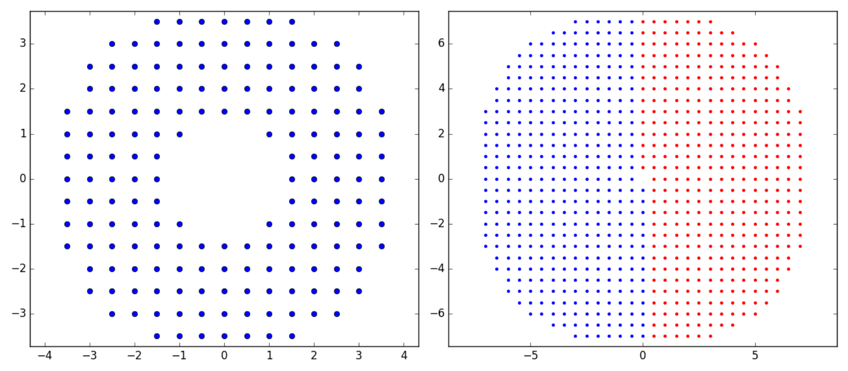
8-m aperture continuous aperture

A different way to look at AO-corrected images



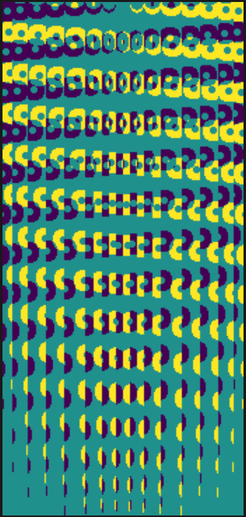
8-m aperture continuous aperture
discretized into a 172-aperture redundant interferometer

A different way to look at AO-corrected images



8-m aperture continuous aperture
discretized into a 172-aperture redundant interferometer
forming 366 distinct baselines
A is a 366x172 matrix : $\Phi = \Phi_0 + A \cdot \varphi$

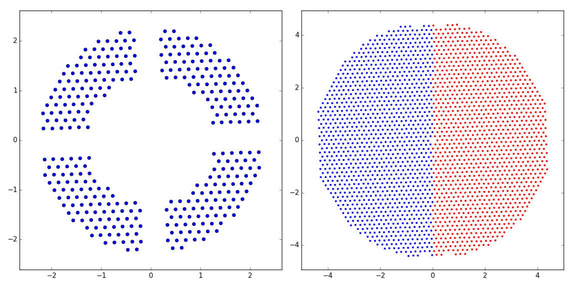
The properties of A

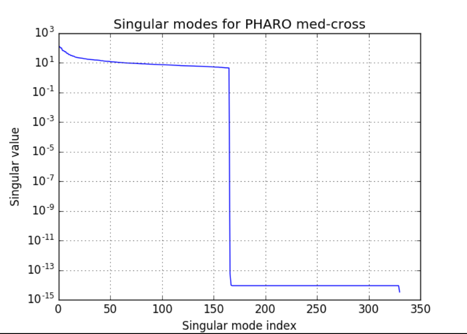


- A is rectangular, sparse
- only contains -1 and +1 (in addition to zeros)
- SVD $\rightarrow$ rank and size of null space
- but in general:
- if the aperture is symmetric:
  - rank(A) = n_A / 2
  - $n_K = n_{UV} - n_A / 2$
- if the aperture is not symmetric
  - rank(A) = n_A - 1
  - $n_K = n_{UV} - n_A - 1$

Since $n_{UV} > n_A$: there is always a kernel. A telescope with a symmetric aperture will result in more kenels.

The properties of the telescope



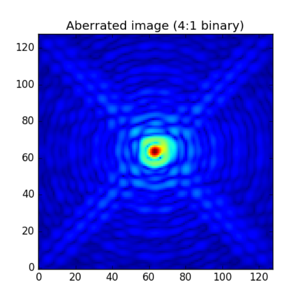


$$\mathbf{A} = \mathbf{U} \cdot \Sigma \cdot \mathbf{V}^T$$

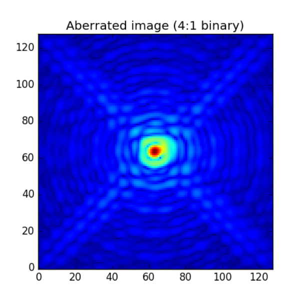
$$\mathbf{U}^T \cdot \mathbf{A} = \Sigma \cdot \mathbf{V}^T$$

The columns of U associated to 0 singular values form the kernel K

A good observable

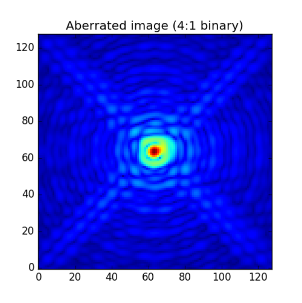


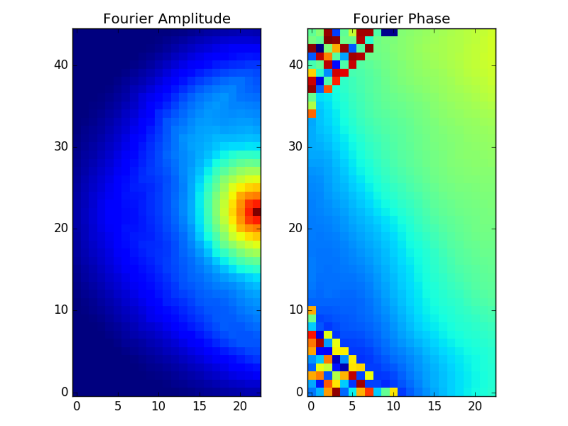
A good observable



FT

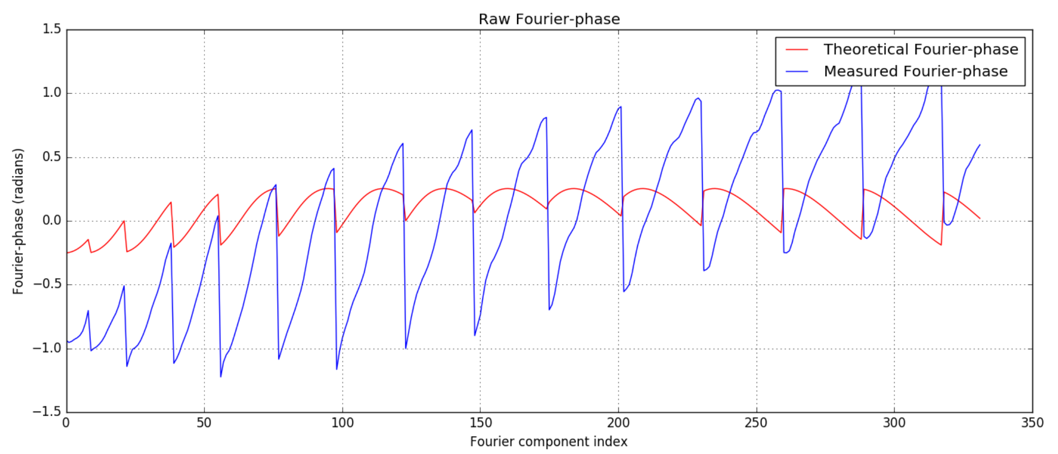
A good observable





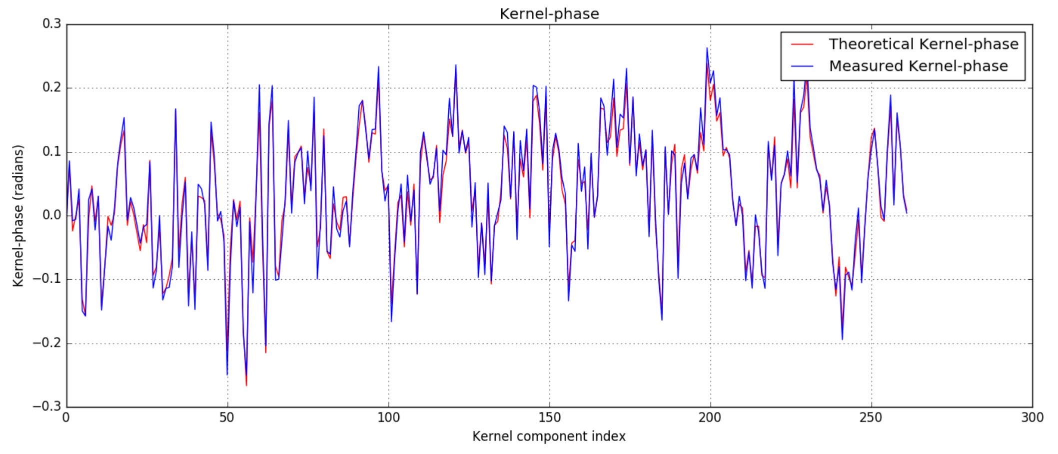
phase: remains dominated by aberrations

A good observable



the raw phase Φ: useless information

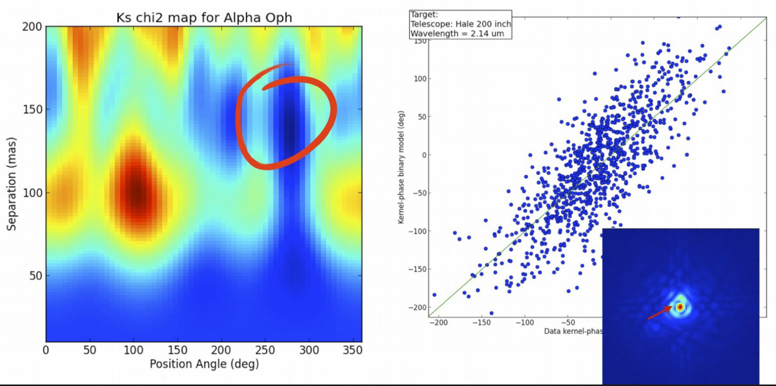
A good observable



kernels $K \cdot \Phi$: 100 % usable information!

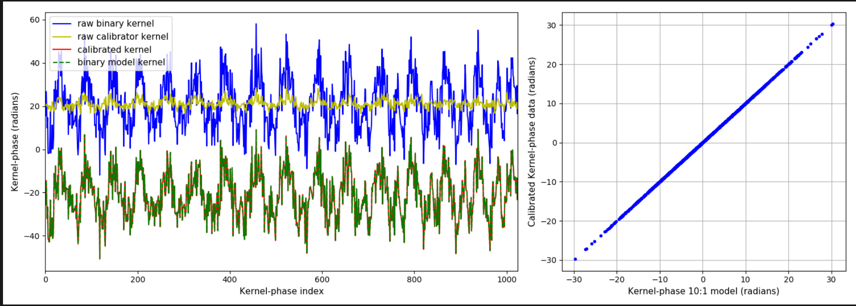
Model-fitting

- The "simplest" scenario: binary detection.
- 3-parameter model: ρ, θ, c (separation, P.A., contrast)
- χ^2 minimization



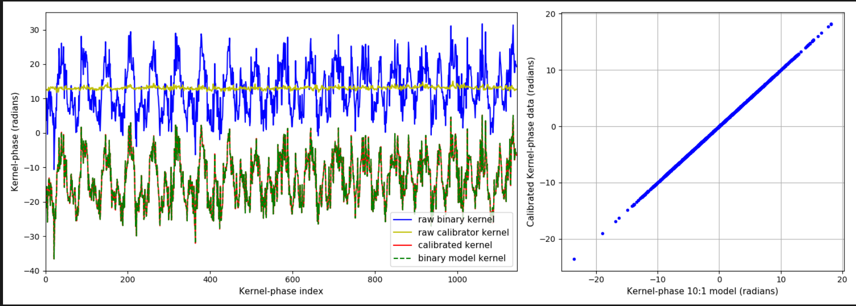
$\rightarrow$ Interferometric Image reconstruction

Today's challenges



- Better description of the aperture
  - Discretization strategies
  - Variable local transmission
- The ability to handle unfriendly scenarios:
  - saturated data
  - larger amplitude aberrations
  - difficult detector cosmetic effects

Today's challenges



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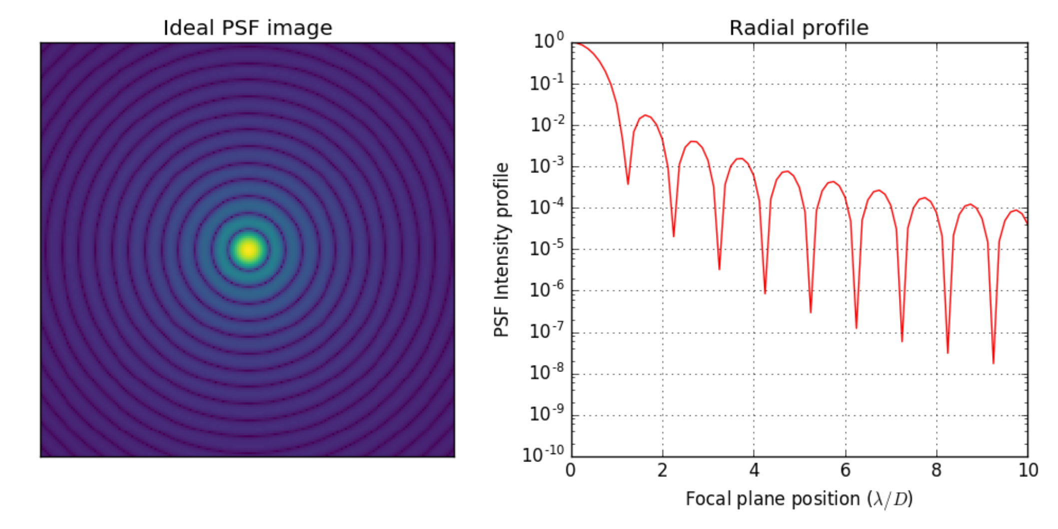
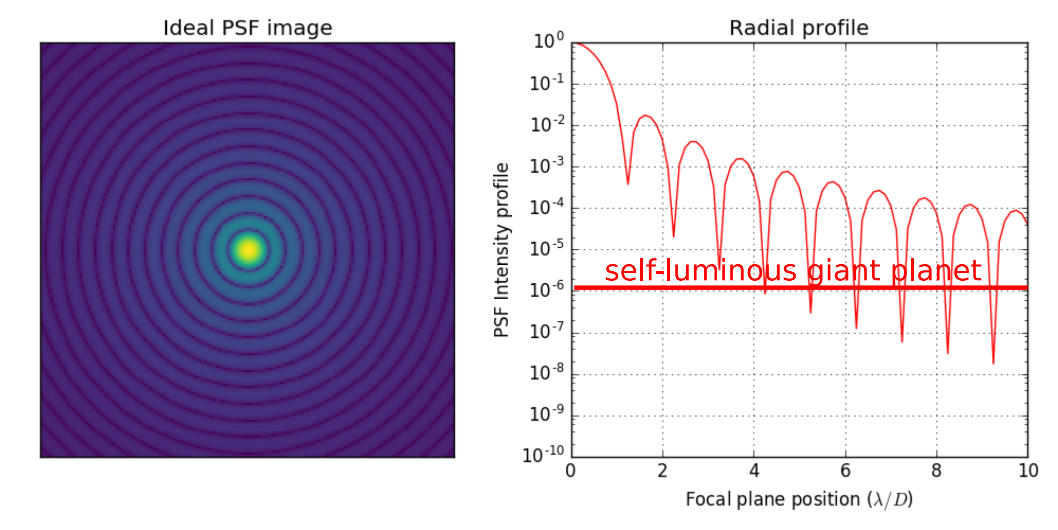
Outline

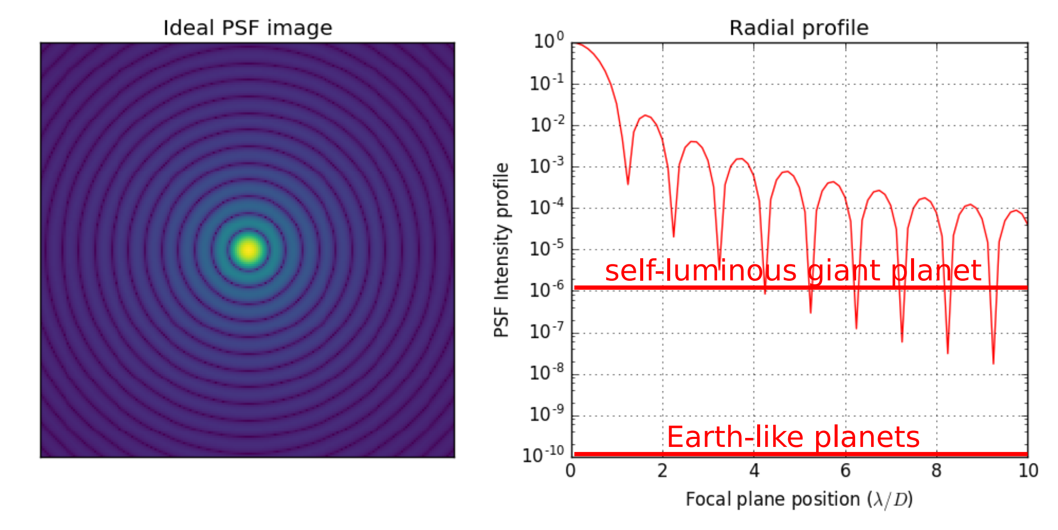
The high-contrast problem



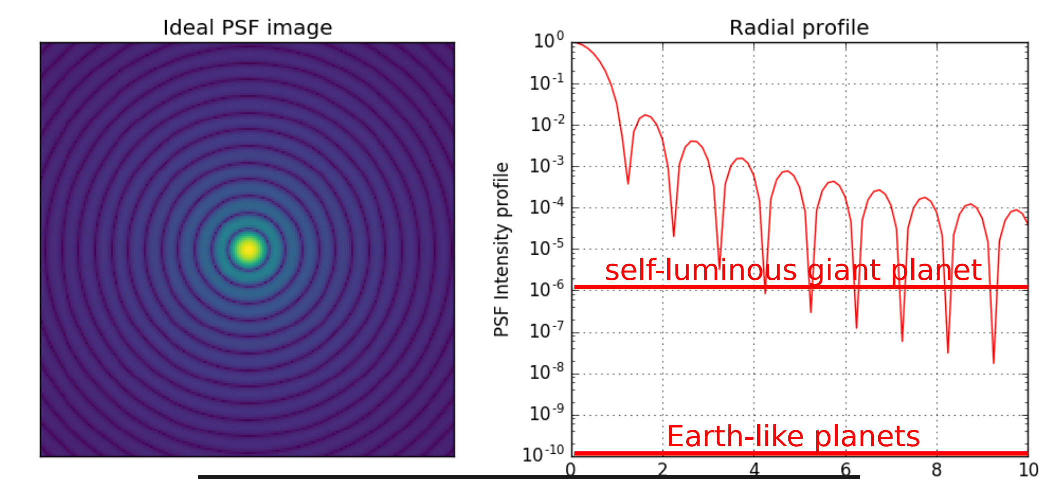
The high-contrast problem



The high-contrast problem



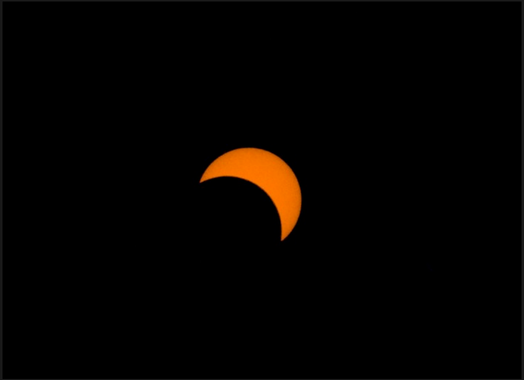
The high-contrast problem



Imaging telescopes are terrible for this job!!

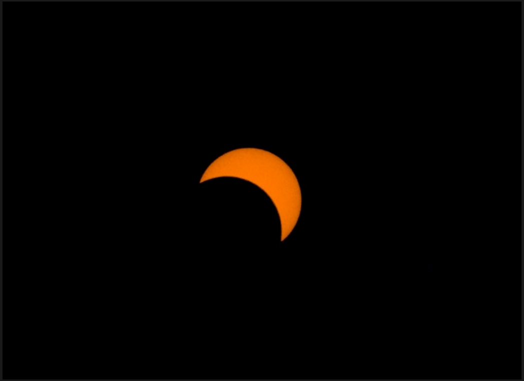
Coronagraphy

Coronagraphy



[Image by O. Lardière]

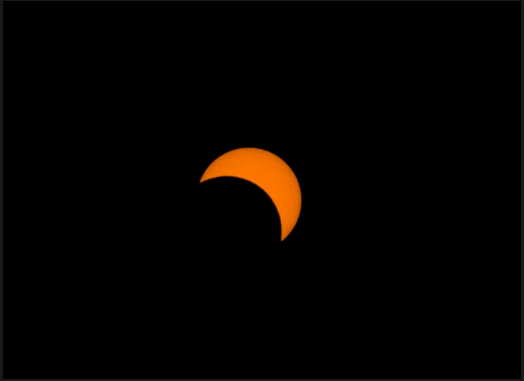
Coronagraphy



[Image by O. Lardière]

[Image by O. Lardière]

Coronagraphy

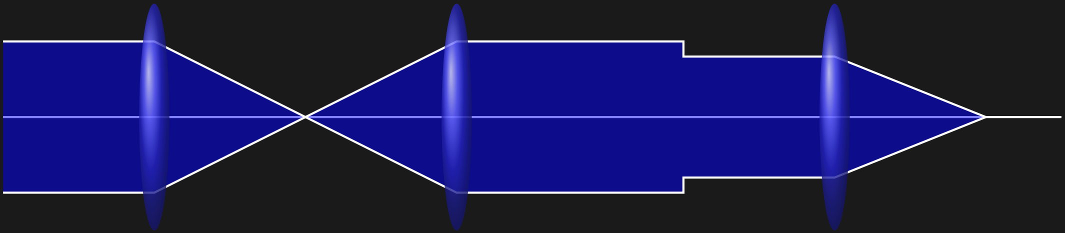


[Image by O. Lardière]

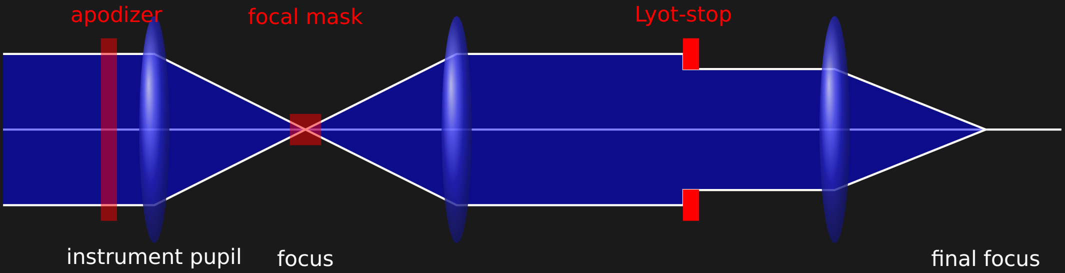
[Image by O. Lardière]

Optically replicate the eclipse phenomenon

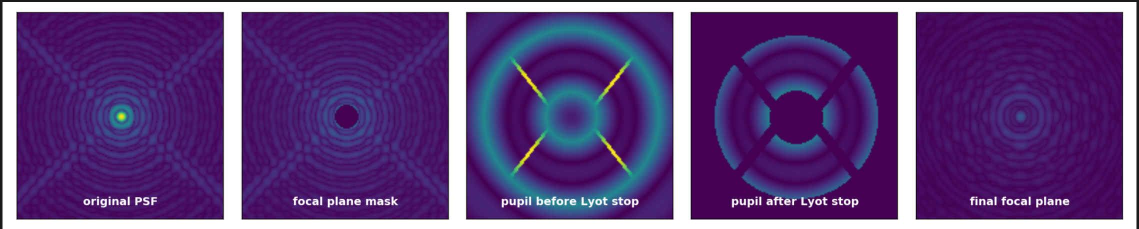
Possible implementation

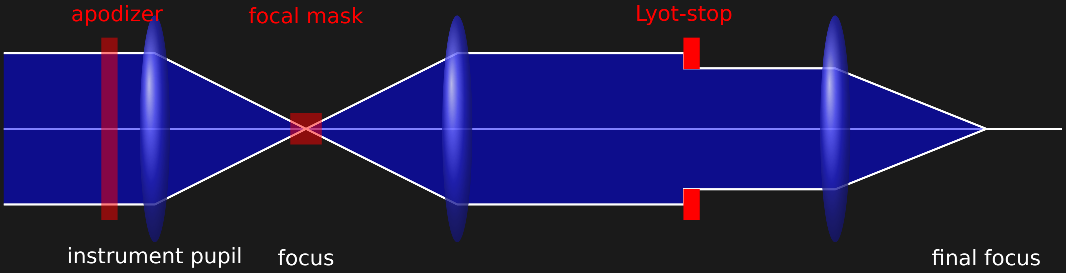


Possible implementation

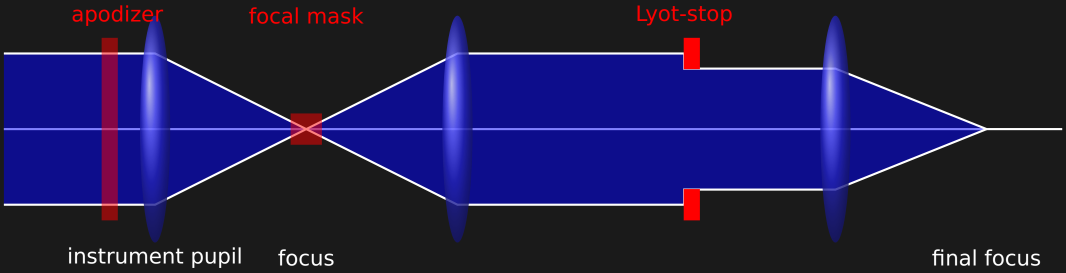


Possible implementation

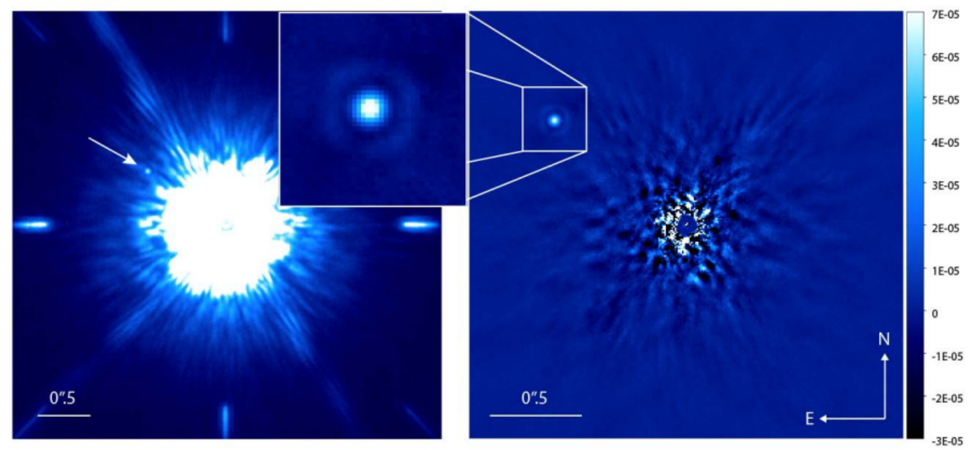




Possible implementation



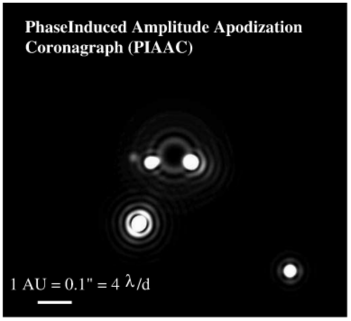
State of the art



Post-processing (partly) saves the day

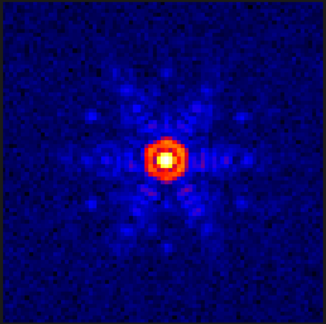
The high-contrast imaging dual challenge

CORONAGRAPHY



Photon noise ✓
Phase noise ✗

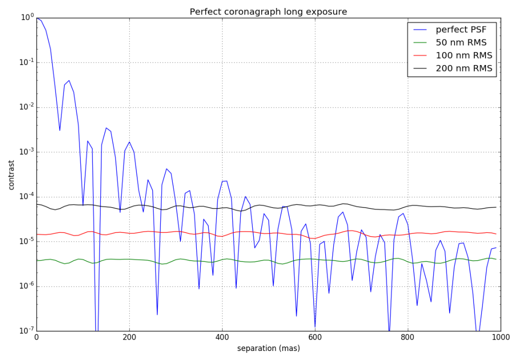
KERNEL



Photon noise ✗
Phase noise ✓

Can we combine the two approaches?

The trick is. . .



The ideal coronagraph

In the presence of a high-contrast device, errors are quadratic: $c \sim (2\pi\sigma/\lambda)^2$

- PSF is not translation invariant
- the problem is non-linear
- the problem is degenerate
- lots of covariance

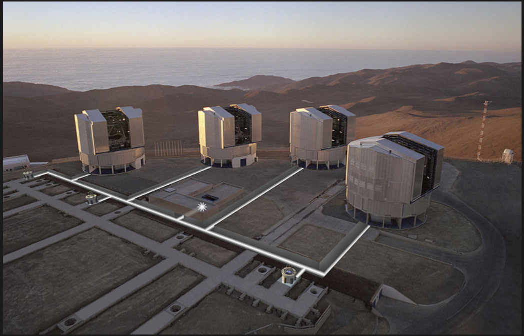
kernel-coronagraphy is a high-dimension problem that is just hard to write

Simplify the problem by going sparse!

How simple?

- Even number of aperture for destructive interferences
- At least three apertures in order to produce a closure

$\rightarrow$ Four sounds abour right!



[VLTI [Credit: ESO]]

Nulling... in theory

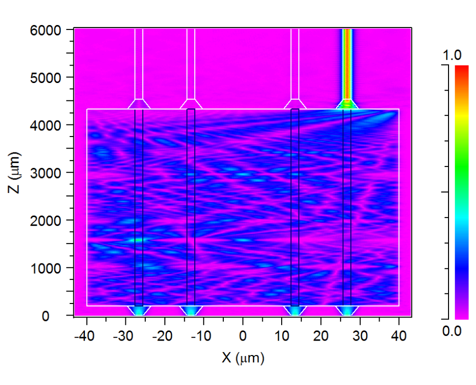
$$\mathbf{N} = 0.5 \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

- Four input beams
- One bright output
- Three dark outputs

Photons of off-axis sources coupled in the dark outputs

Still sensitive to perturbations.

4x4 nuller



Integrated optics technology option
MMI design by Harry-Dean Kenchington Goldsmith, ANU

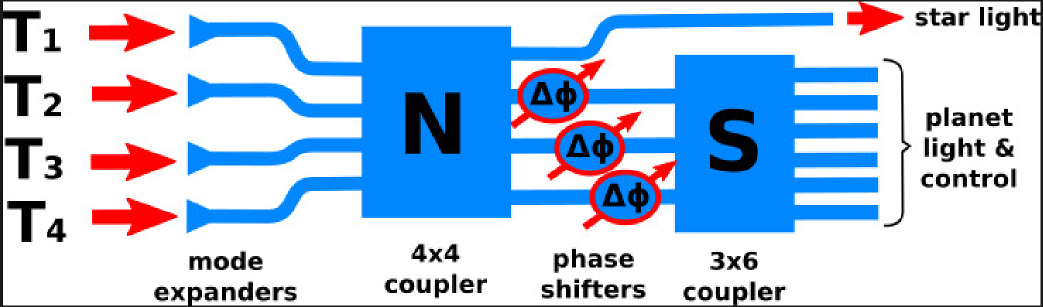
System response to perturbation is quadratic

- Nuller error dominated by second order errors
- Modify the instrument design to produce kernels of these perturbations

$$\delta \mathbf{I} = \mathbf{A} \times \left[\frac{\partial^2 \mathbf{I}}{\partial \varphi_1^2}, \frac{\partial^2 \mathbf{I}}{\partial \varphi_2^2}, \frac{\partial^2 \mathbf{I}}{\partial \varphi_3^2}, \frac{\partial^2 \mathbf{I}}{\partial \varphi_1 \partial \varphi_2}, \frac{\partial^2 \mathbf{I}}{\partial \varphi_1 \partial \varphi_3}, \frac{\partial^2 \mathbf{I}}{\partial \varphi_2 \partial \varphi_3} \right]^T . \quad (1)$$

- record a new 2^{nd} order perturbation response matrix **A**
- and find a kernel for this matrix

Nuller → Kernel-nuller

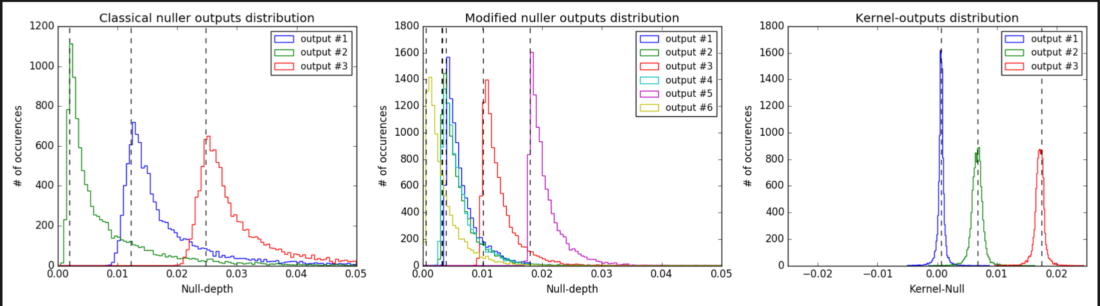


The innovation: a 2^{nd} stage, a scrambling unit that:

- makes the outputs respond to perturbation in an asymmetric manner
- builds kernels: observables robust against second order piston errors

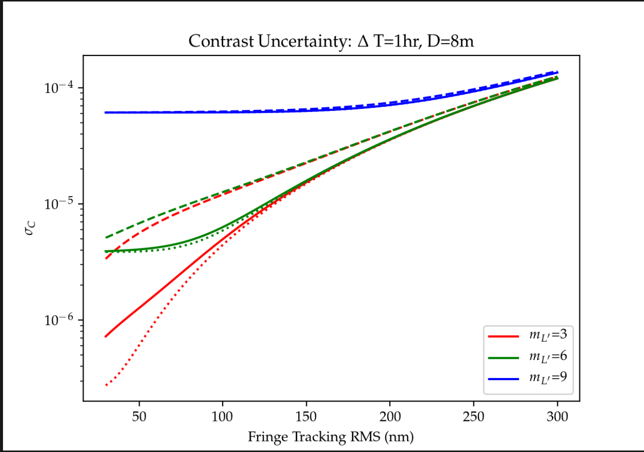
Kernel-nulled outputs: robustness is possible

In the presence of piston residual errors



kernels filter out second order errors

VIKiNG contrast detection limits



Phase induced contrast leaks:
c $\sim (2\pi\sigma/\lambda)^3$

5-σ L-band (4-UTs) contrast detection limits.

Performance depends on:

- cophasing stability
- star magnitude (compete with sky background)
- injection stability (AO correction)

Planets are within our grasp

Outline

A new playground for Artificial Intelligence?

Many signal processing considerations:

- discretization
- detection (statistical tests)
- non-linear problems
- inverse problems (interferometric image reconstruction)
- instrument design guided by signal processing ideas

A playground for Artificial Intelligence?

- adaptive optics predictive control?
- AO telemetry and images for better post-processing?
- large observing multi-epoch campains (lots of data)
- telescope observing scheduling
- interferometric beam combiner design?

KERNEL