ONERA

THE FRENCH AEROSPACE LAB

www.onera.fr

Differential ray tracing and optimal design applications

Jean-Baptiste Volatier

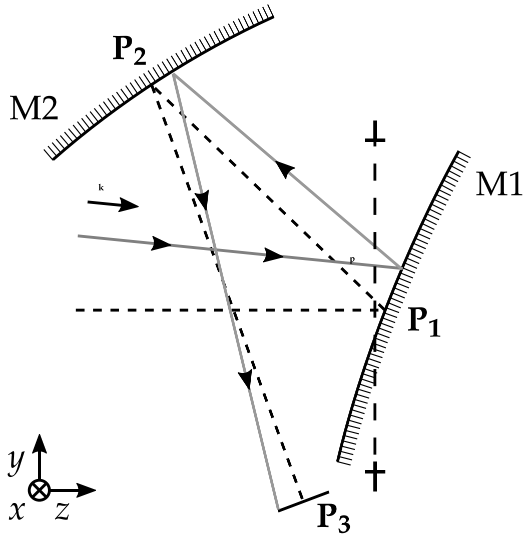
ONERA
THE FRENCH AEROSPACE LAB

What is a ray?

- The ray is the fundamental abstraction of geometrical optics
- It approximates the light path through an optical system
- From ray tracing one can simulate an optical system and compute optical performance metrics of interest

What is differential ray tracing ?

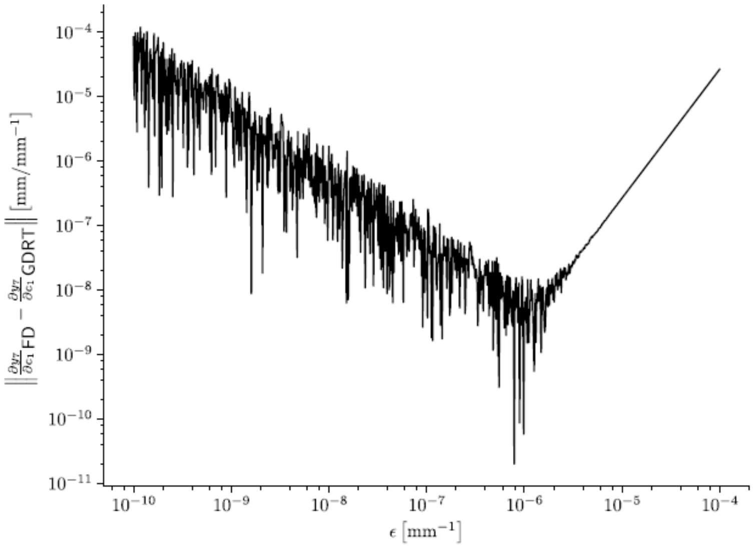
- **Differential calculus of the sensitivity of a ray to changes:**
  - In the optical system geometry (e. g. curvature of a lens surface)
  - In the parameters of the ray itself (e. g. point of origin of the ray on a scene)
- **No finite difference approximation !**
- **Topics of today:**
  - Gradient-based optimization
  - Construction of optical surfaces by integration



Gradient-based optimization

Finite difference approximation, why not ?

- **Finite difference approximation is innacurate**
  - optical system → non linear
- **Finite difference is slow**
  - when many parameters are considered



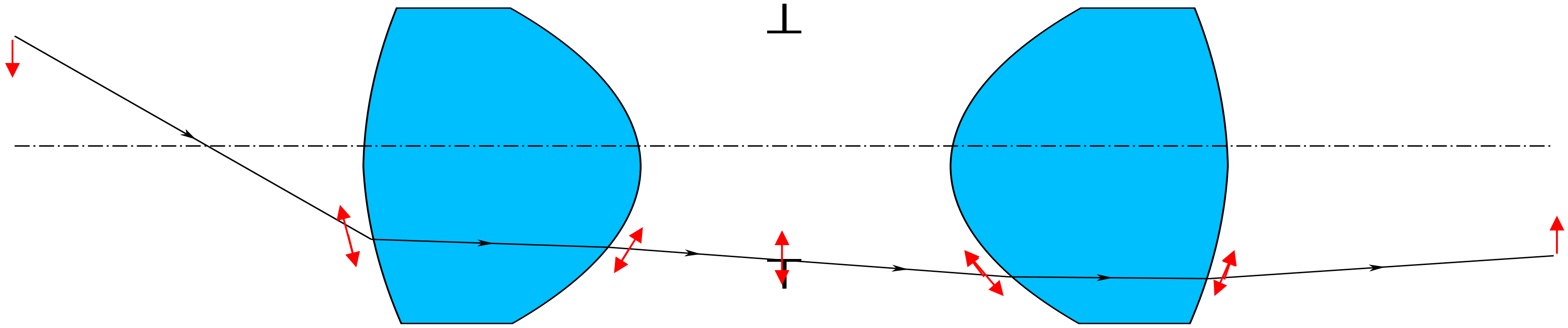
Volatier, Jean-Baptiste, Álvaro Menduiña-Fernández, and Markus Erhard. 'Generalization of Differential Ray Tracing by Automatic Differentiation of Computational Graphs'. *Journal of the Optical Society of America A* 34, no. 7 (1 July 2017): 1146. https://doi.org/10.1364/JOSAA.34.001146.

State-of-the-art of differential ray tracing

Not a new idea !

- 1960 – spherical lenses
- 1990 – extension to rotationally symmetric surfaces
- 2000 – extension to prisms and 2^{nd} order differentiation
- 2017 – general formalism applicable to all shapes

Differentiation of Fermat Path Principle

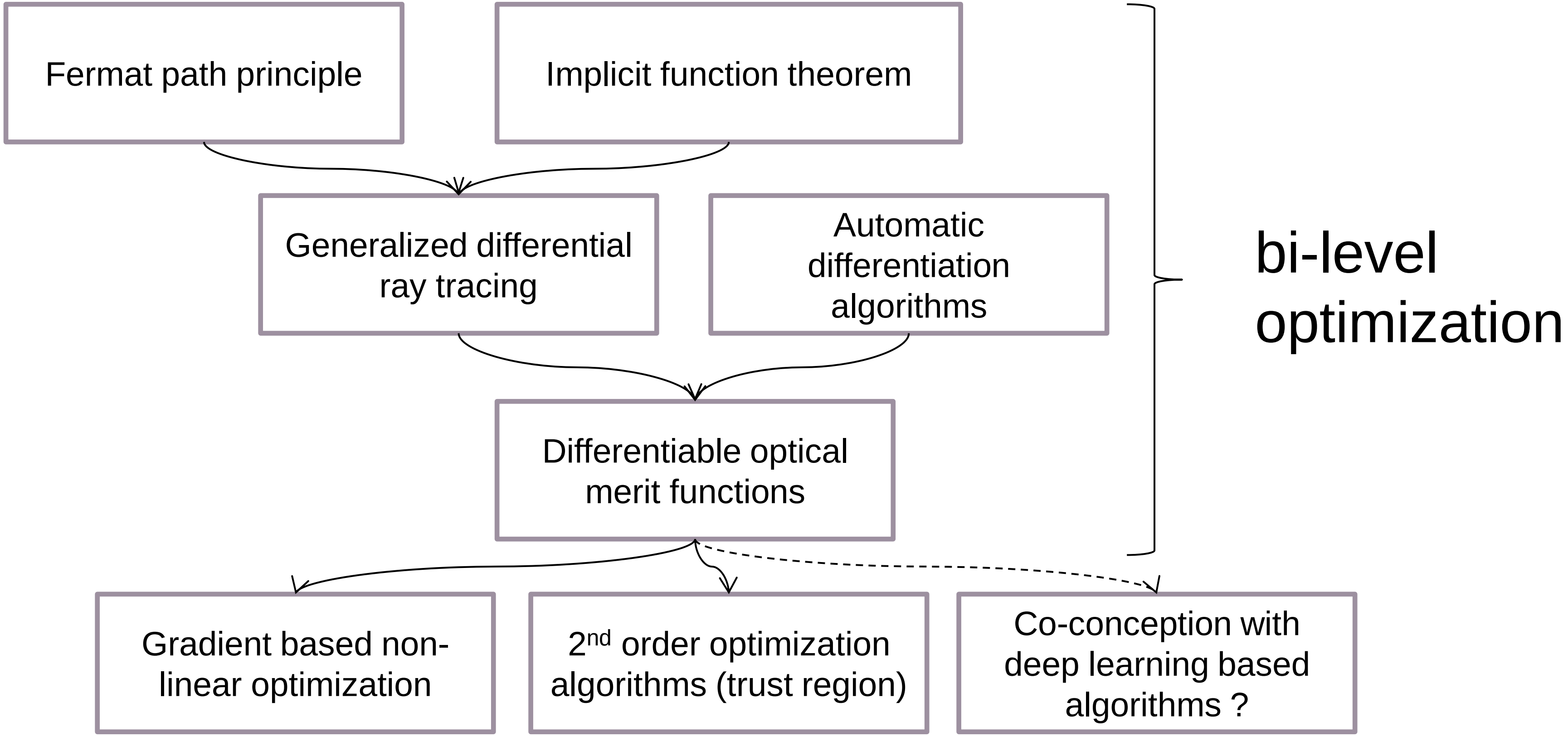


Optical path length: $\boldsymbol{L}$

Fermat error function: $\mathbf{F} = \nabla \boldsymbol{L} = \mathbf{0}$

First order perturbation : $\nabla \boldsymbol{F}.\boldsymbol{\varepsilon} = \mathbf{0}$

Gradient-based non-linear optimization



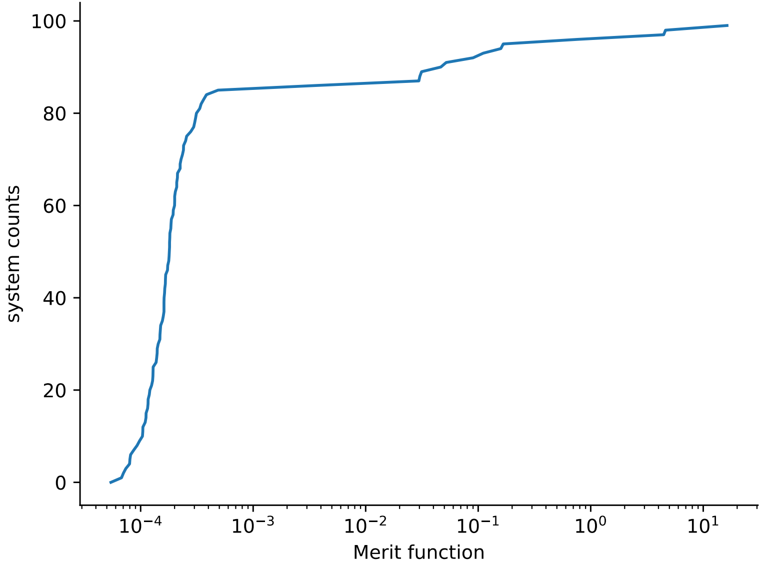
Current work @ ONERA

- Implementation in Julia of algorithm [1]
- Using Forward Differentiation for the automatic differentiation part [1]

[1] Volatier, Jean-Baptiste, Álvaro Menduiña-Fernández, and Markus Erhard. ‘Generalization of Differential Ray Tracing by Automatic Differentiation of Computational Graphs’. *Journal of the Optical Society of America A* 34, no. 7 (1 July 2017): 1146. https://doi.org/10.1364/JOSAA.34.001146.
[2] Revels, Jarrett, Miles Lubin, and Theodore Papamarkou. ‘Forward-Mode Automatic Differentiation in Julia’. *ArXiv:1607.07892 [Cs]*, 26 July 2016. http://arxiv.org/abs/1607.07892.

Preliminary results

- **Using Houllier [1] refractive case study**
- **IPOPT [2] solver without hessians**
- **500 solver iterations max (~5000/500 function/gradient eval.)**
- **To be continued…**
  - Not that good in the « end-game » cases



[1] Houllier, Thomas, and Thierry Lépine. ‘Comparing Optimization Algorithms for Conventional and Freeform Optical Design’, n.d., 18.
[2] Wächter, Andreas, and Lorenz T. Biegler. ‘On the Implementation of an Interior-Point Filter Line-Search Algorithm for Large-Scale Nonlinear Programming’. *Mathematical Programming* 106, no. 1 (1 March 2006): 25–57. https://doi.org/10.1007/s10107-004-0559-y.

Freeform optics

- **Freeform optics → design with shapes that:**
  - Are not spheres
  - Do not have an axis of symmetry
  - Usually keep a plane of symmetry
- **Can be adjusted by non-linear optimization**
  - But how to choose the surface parametrization ?
- **Or resolution of a PDE**

Design of optical surfaces by integration

Wassermann, Wolf 1948

On the Theory of Aplanatic Aspheric Systems

By G. D. WASSERMANN * and E. WOLF †

H. H. Wills Physical Laboratory, University of Bristol

* Now at King's College, University of Durham

† Now at the Observatories, University of Cambridge

MS. received 21st February 1948; read before Optical Group 17th December 1948

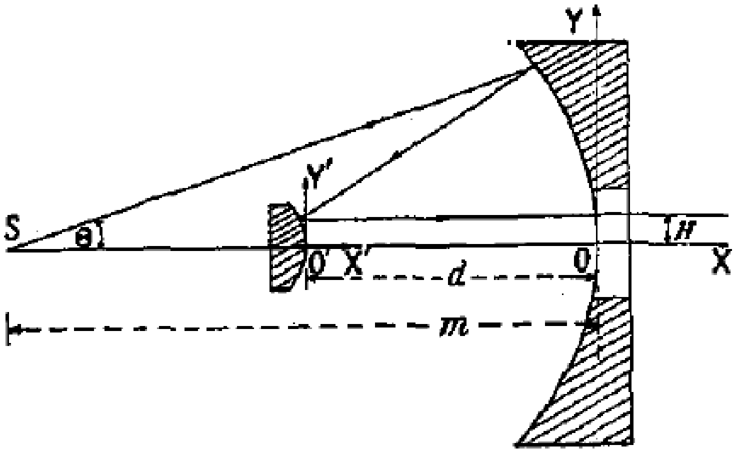
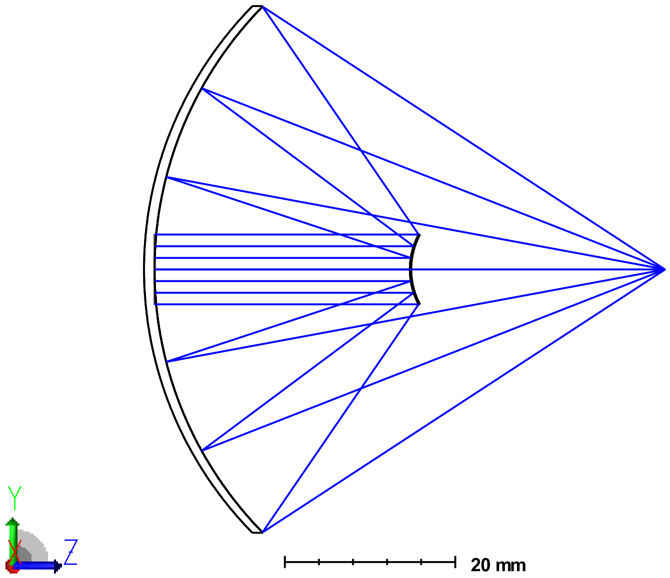
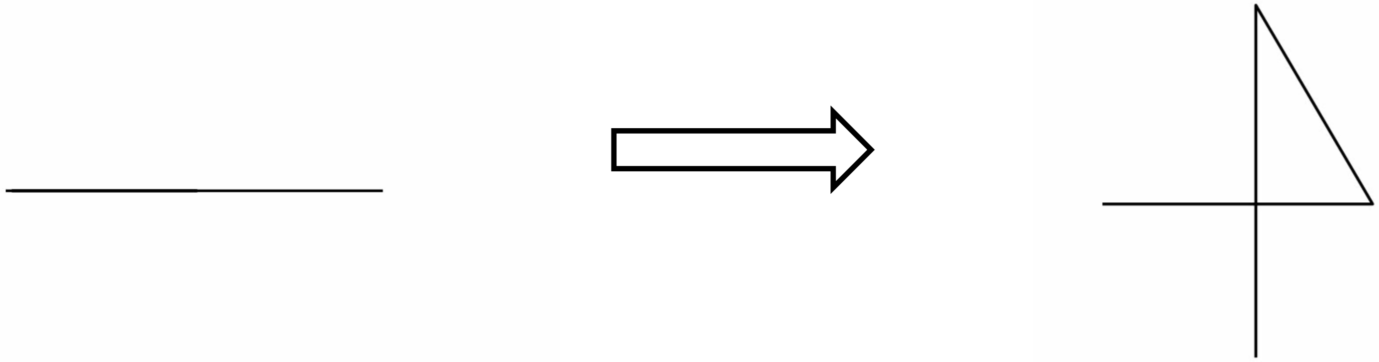


Figure 3.

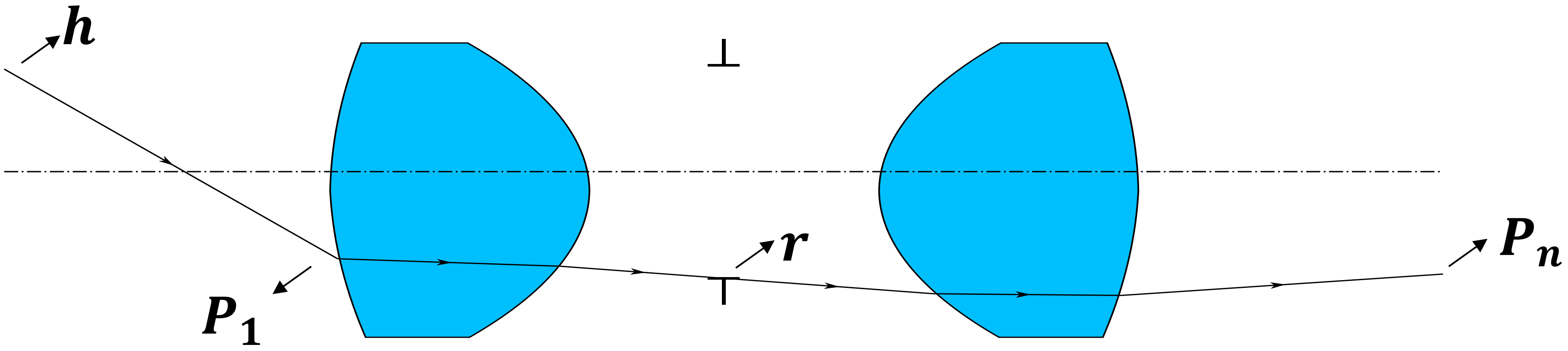


Breaking the axial symmetry (planar symmetry maintained)



Volatier, Jean-Baptiste, and Guillaume Druart. ‘Differential Method for Freeform Optics Applied to Two-Mirror off-Axis Telescope Design’. *Optics Letters* 44, no. 5 (1 March 2019): 1174.

The ray tracing function



Phase space: $\phi = (h; r) \in \mathbb{R}^4$

Raytracing function: $\phi \mapsto (P_1, \dots, P_i, \dots, P_n)$

Point on a surface: $\begin{array}{c} \mathbb{R}^2 \to \mathbb{R}^3 \\ w \mapsto P \end{array}$

ONERA THE FRENCH AEROSPACE LAB

Parameter spaces Example 1: Sag

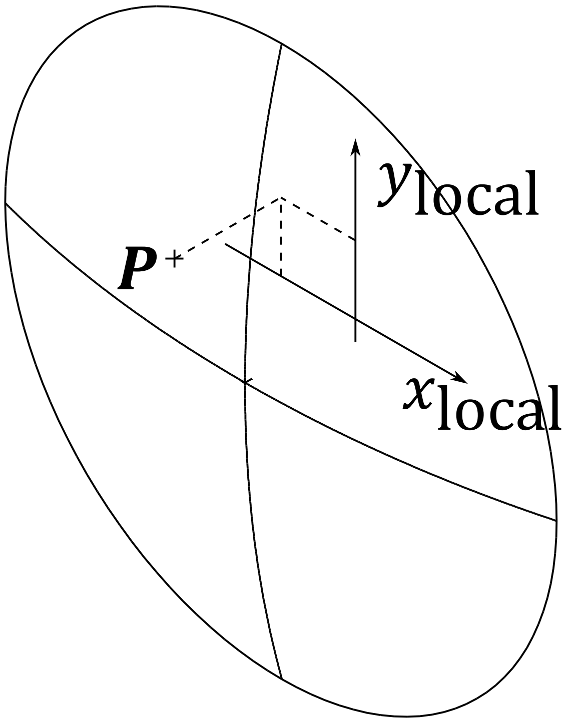
Point in local coordinates:

$$\boldsymbol{P}_{\text{local}} = \begin{pmatrix} x_{\text{local}} \\ y_{\text{local}} \\ \text{sag}(x_{\text{local}}, y_{\text{local}}, c) \end{pmatrix}$$

Affine transformation to global coordinate system:

$$\boldsymbol{P} = \boldsymbol{A}\boldsymbol{P}_{\text{local}} + \boldsymbol{\Delta}$$

Parameter vector: $w = (x_{\text{local}}, y_{\text{local}})$



Parameter spaces Example 2: Bézier surface

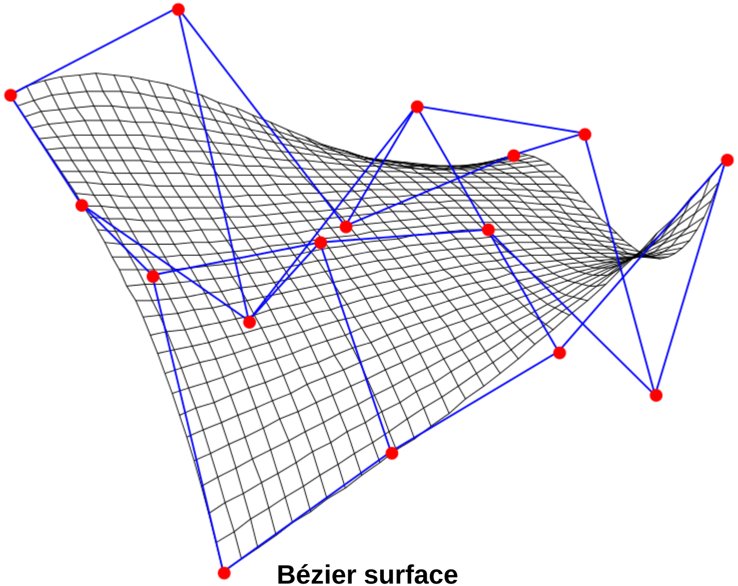
Bézier surface:

$$\boldsymbol{p}(\boldsymbol{u},\boldsymbol{v}) = \sum_{i=0}^{n}\sum_{j=0}^{m} B_i^n(u)B_j^m(v)\boldsymbol{k}_{\boldsymbol{i},\boldsymbol{j}}$$

Parameter vector:

$$\boldsymbol{w} = (u, v)$$

Parameter vector does not correspond to a point on a plane in physical space



Bézier surface
Wojciech Mula at Polish Wikipedia [CC BY-SA 3.0]

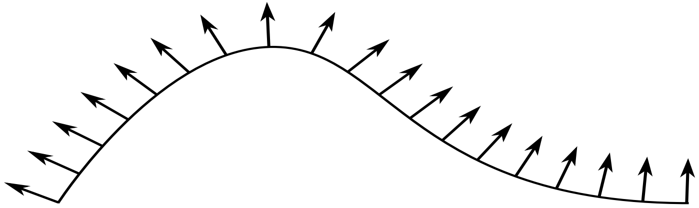
Solution to surface inconsistencies : Dimensional bottleneck

Dimensions	$\mathbb{R}^4$	$\rightarrow$	$\mathbb{R}^2$	$\rightarrow$	$\mathbb{R}^3$
Space	Phase	$\rightarrow$	Parameter	$\rightarrow$	Physical
functions	$\boldsymbol{\phi}$	$\mapsto$	$\boldsymbol{w_i}$	$\mapsto$	$\boldsymbol{P_i}$

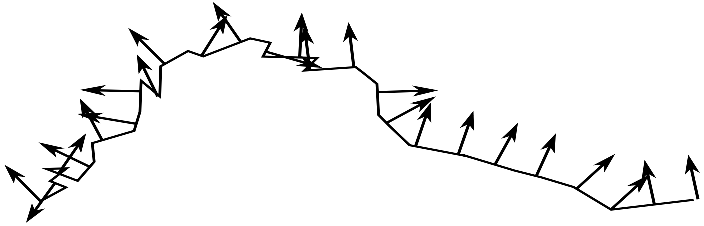
$$\left[\frac{\partial \boldsymbol{P_i}}{\partial \boldsymbol{\phi}}\right] = \left[\frac{\partial \boldsymbol{P_i}}{\partial \boldsymbol{w_i}}\right]\left[\frac{\partial \boldsymbol{w_i}}{\partial \boldsymbol{\phi}}\right] \quad \Rightarrow \quad \operatorname{rank}\left[\frac{\partial \boldsymbol{P_i}}{\partial \boldsymbol{\phi}}\right] \leq 2$$

How to keep surfaces consistent ?

- Construction – iteration method [1]
- Parameter space



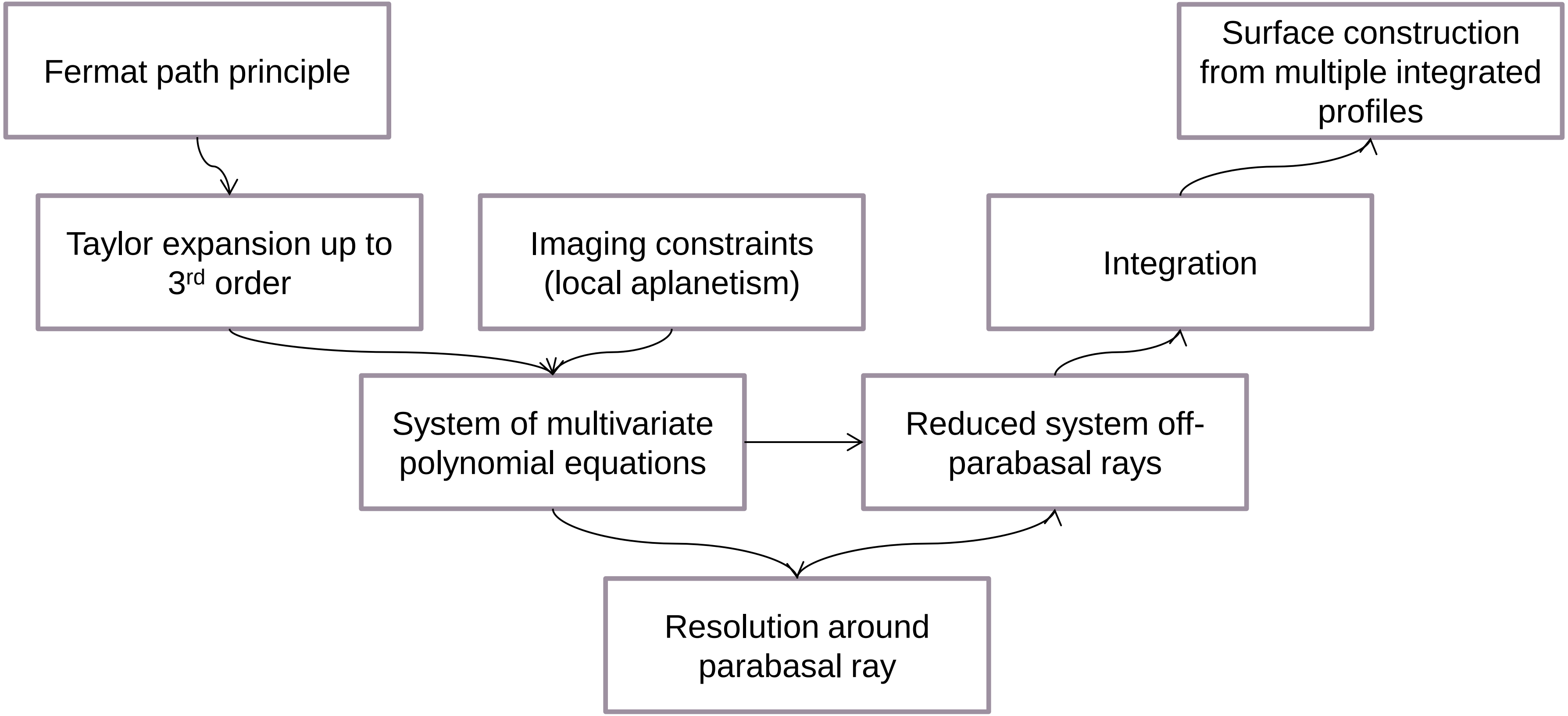
Consistent surface



Inconsistent surface

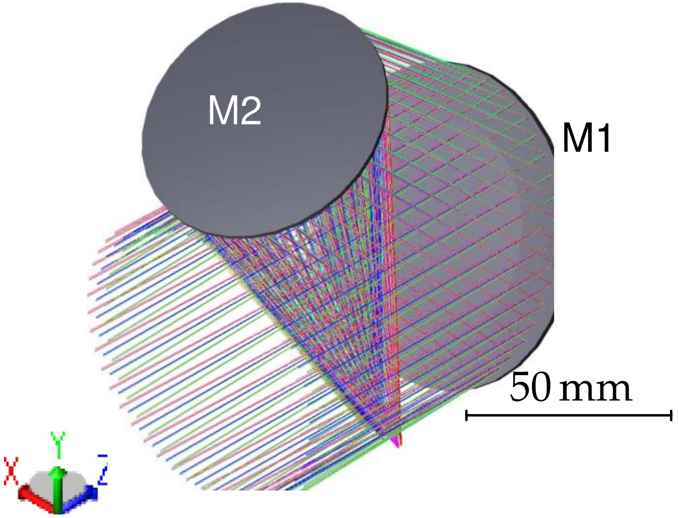
[1] Yang, Tong, Guo-Fan Jin, and Jun Zhu. ‘Automated Design of Freeform Imaging Systems’. Light: Science & Applications 6, no. 10 (6 October 2017): e17081.

Integration of surface profiles

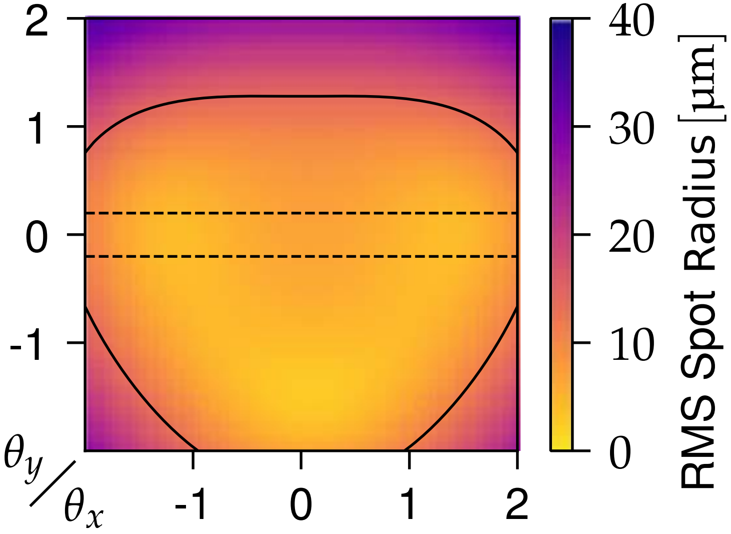


Application to off-axis two-mirror design

Focal length: 150mm
pupil size: 70mm
FoV: $4 \times 0.2^\circ$



Refocused performance



ONERA
THE FRENCH AEROSPACE LAB

Conclusion

Conclusion and outlook

- **Optical design problems are differentiable**
  - Have they other interesting properties?
  - Can be used for co-conception
- **Freeform surfaces can be constructed by PDE integration**
  - Can we go beyond aplanetism?
  - Can we extend to > 2 surfaces?

ONERA
THE FRENCH AEROSPACE LAB