Asplund's Metrics Useful for Images Acquired under Variable Lighting

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- Images acquired under unstable lighting conditions are very difficult to process.
- For example, such a situation occurs for video-surveillance images of streets, highways, military or industrial sensitive sites, aerial or submarine images …
  and also:
- Images acquired “in transmission” (observed object between the source and the sensor): microscopy, tomography, radiography, scanner…, in which the opacity and the thickness of the object may be variable.
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- In order to overcome the drawbacks generated by a variable lighting, we propose to develop our image processing algorithms inside the LIP (Logarithmic Image Processing) framework.

- We will see that such a framework permits to develop tools and algorithms presenting a lot of significant advantages compared to the « classical » ones.

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- The most significant properties of the LIP Model:
  - It is based on the «Transmittance Law», which gives it a strong physical justification
  - The laws defined inside the Model (addition of two images and homothety of an image) possess strong mathematical properties due to the fact they give a Vector Space structure to the space of images
  - Moreover, Brailean (*) has established that the Model is consistent with the Human Visual System

Recalls on the LIP Model

Notations: A grey level image is a function defined on the spatial domain $D$ with values in the grey scale $[0, M]$:

$$f : D \subset \mathbb{R}^2 \rightarrow [0, M] \subset \mathbb{R}$$

- **Definition of the Logarithmic Addition Law**

The addition $f \triangle g$ of two images $f$ and $g$ is interpretable (for images acquired in transmission) as the superimposition of the obstacles (objects) generating respectively $f$ and $g$. 

\[ \begin{align*} 
    & S \rightarrow \text{Source} \\
    & S \rightarrow \text{Sensor} \\
    & S \rightarrow f \\
    & S \rightarrow g \\
    & S \rightarrow f \triangle g 
\end{align*} \]
Recalls on the LIP Model

Definition of the Logarithmic Addition Law
– It has been established that

\[ f \bigtriangleup g = f + g - \frac{f \cdot g}{M} \]  \hspace{1cm} (1)

Remark: the opposite of \( f \) and the subtraction of \( f \) and \( g \) have also been defined according to the formulas:

\[ \bigtriangledown f = \frac{-f}{1 - \frac{f}{M}} \quad \text{and} \quad f \bigtriangledown g = \frac{f - g}{1 - \frac{g}{M}} \]  \hspace{1cm} (2)

Warning: within the LIP model, 0 corresponds to the “white” extremity of the grey scale, which means to the source intensity, i.e. when no obstacle (object) is placed between the source and the sensor.

The other extremity \( M \) is a limit situation where no element of the source is transmitted (black value). For 8-bits images, \( M = 2^8 = 256 \).
Recalls on the LIP Model

Variation of the source intensity

An image $f$ represented in the scale $[0, M]$ becomes $f \uplus C$ in the scale $[C, M]$, which corresponds to an attenuation of $f$

$\Rightarrow$ on the opposite,

0 appears as a « negative » grey level related to $[C, M]$ and negative grey levels appear as light intensifiers
Recalls on the LIP Model

Definition of the Logarithmic Scalar Multiplication

- For every $\lambda \in \mathbb{R}$

$$\lambda \diamond f = M - M \left(1 - \frac{f}{M}\right)^\lambda$$

Remark: such a multiplication consists of stacking $f$ upon itself $\lambda$ times $\Rightarrow$ the resulting image is darker (resp. brighter) than $f$ when $\lambda > 1$ (resp. $\lambda < 1$) and obviously unchanged when $\lambda = 1$

To conclude these recalls, the two laws $\bigtriangleup$ and $\bigtriangledown$ equip the space of grey level functions with a Vector Space structure, which gives access to a countless set of tools and concepts available in this kind of spaces (for example the logarithmic interpolation between two images or the scalar product of two images, which represents in fact a correlation coefficient between them).
What can we expect from the LIP framework?

- We can define new notions of contrast, physically founded and interpretable:
  - The Logarithmic Additive Contrast between two points \((x, y)\):

\[
C_{(x,y)}(f) = \text{Max}(f(x), f(y)) \triangle \text{Min}(f(x), f(y))
\]  

\[C_{(x,y)}(f) = \frac{|f(x) - f(y)|}{1 - \frac{\text{Min}(f(x), f(y))}{M}}
\]  

Such a contrast represents the grey level we must add to the brightest pixel \((\text{Min}(f(x), f(y)))\) to get the darkest one \((\text{Max}(f(x), f(y)))\). Using the subtraction formula (2) yields to the explicit expression:
What can we expect from the LIP framework?

**Contour detection:**

We can compute the LIP contrast between a pixel \( x \) and each of its 8 neighbors and affect to \( x \) the maximal one:

- Initial image of a well
- Contour detection (classical Sobel)
- Contour detection (LIP contrast)
What can we expect from the LIP framework?

**Image enhancement**: when we subtract (resp. add) a constant (i.e. a grey level) to an image in the LIP sense, we brighten (resp. darken) it.
What can we expect from the LIP framework?

Image stabilization (may be done at camera speed of 25 im./s)

a) Variable lighting
b) Stabilized images
c) Corresponding histograms
What can we expect from the LIP framework?

Exposure time simulation (LIP addition/subtraction of a constant)

a) Image f: “Laboratory” acquired with exposure time = 10ms
b) Image g: “Laboratory” acquired with exposure time = 100ms
c) Starting from a), simulation of an exposure time of 100ms
What can we expect from the LIP framework?

Exposure time simulation

a) Label acquired at 30ms (motionless wheel)
b) Label acquired at 30ms (wheel rotating at 6.4 revolutions per second)
c) Rotating label acquired at 1ms
d) Simulation of an exposure time of 30ms starting from image c)
What can we expect from the LIP framework?

Texture evaluation independently of the lighting

a) Initial image
b) LIP additive maximal contrast applied to a)
c) Sobel operator applied to a)
What can we expect from the LIP framework?

Defining novel metrics (distances between images)

Let us recall formula (4):

\[ C_{(x,y)}(f) = \text{Max}(f(x), f(y)) \oblong \text{Min}(f(x), f(y)) \]

defining the Logarithmic Additive Contrast between two points \( x \) and \( y \) of an image \( f \)

In the same way, we can compute the Logarithmic Additive Contrast between two images \( f \) and \( g \) at the same point \( x \) according to:

\[ C_{(x)}(f, g) = \text{Max}(f(x), g(x)) \oblong \text{Min}(f(x), g(x)) \]  

(6)

This is the first step for defining a novel distance between \( f \) and \( g \)
What can we expect from the LIP framework?

Defining novel metrics (distances between images)

In fact, we can get a Logarithmic distance between $f$ and $g$ on a ROI by computing the average value or the maximal value of $C_{(x)}^\triangle(f, g)$ when $x$ varies in the ROI.
What can we expect from the LIP framework?

Defining novel metrics (distances between images)

Another approach consists of starting from a little known metric defined on binary shapes: the Asplund’s metric.

A and B being two binary shapes, Asplund proposed to perform a double probing of one shape (A for example) by the other:

We compute the smallest number $\lambda_0$ such that $\lambda_0B$ contains $A$ and the greatest number $\mu_0$ such that $A$ contains $\mu_0B$.

and we define the Asplund’s distance between $A$ and $B$:

$$d_{As}(A, B) = \ln \left( \frac{\lambda_0}{\mu_0} \right)$$
ASPLUND’s Metrics

Multiplicative Asplund Metric

Now we propose to extend this approach to a pair \( (f, g) \) of images:

\[
d_{AS}^\Delta (f, g) = \ln \left( \frac{\lambda_0}{\mu_0} \right)
\]

Note that such a distance does not change when one function is replaced by an homothetic \( \Rightarrow d_{AS} \) is insensitive to lighting variations modelized by \( \lambda \Delta \).

Such a metric performs a “double sided probing” of \( f \) by \( g \)
Example of application: Target tracking

a) Initial image
b) Bright target $t_1$ (a bright brick of the wall)
c) Corresponding Asplund’s map (values of $d_{AS}^\Delta(f|_{D_{t_1}}, t_1)$)
d) Dark target $t_2$ (a dark brick of the wall)
e) Corresponding Asplund’s map (values of $d_{AS}^\Delta(f|_{D_{t_2}}, t_2)$)
Remark: The Multiplicative Asplund’s metric is sensitive to noise.

Such a drawback is shared with other “atomic” metrics like Hausdorff.

We have proposed a solution to limit this effect. It consists of discarding the most penalizing points (see the following Ref.)

More recently, I have introduced a new kind of Asplund’s metric: the \textbf{Additive Asplund’s Metric}. In fact, the approach is quite different: the double sided probing is no more performed thanks to homothetics of the probe, but by applying a LIP addition of a constant:

Given two grey level images $f$ and $g$, we define two real numbers $C_1$ and $C_2$ according to:

\[
C_1 = \text{Inf}\{C, g \bigtriangleup C \geq f\}
\]

\[
C_2 = \text{Sup}\{C, g \bigtriangleup C \leq f\}
\]

where $C$ lies in $\left]-\infty, M\right[$

\[
d_{As}(f, g) = C_1 \bigtriangleup C_2 \\
\in [0, M]
\]
**Advantage of this approach:** We have seen the addition/subtraction of a constant simulates exposure time variation, as well as variation of the source intensity.

⇒ the “additive” double sided probing becomes *insensitive to such variations*

This opens the way to a lot of applications like pattern recognition, target tracking… independently of the lighting conditions

**Recall:** Thanks to the paper of Brailean, the consistency of the LIP operators with Human Vision permits to *process and analyze images acquired in reflexion as a human eye would do.*

Let us begin by a theoretical example:
Consider Lena and a darkened image

Compute the Asplund’s maps of the target on $f$ and $f \Delta 240$: 

$f$

$C \Delta 240$

target (magnified)
ASPLUND’s Metrics

Real case:
Expo: 170ms

Additive
Asplund’s map

garbage can (magnified)

Expo: 50ms
Our goal is not to expose in depth the possible use of Asplund’s metrics.

Nevertheless, I must highlight the following point: When we compute the Asplund’s distances $d_{\text{As}}(f, g)$ and $d_{\text{As}}(f, g)$, we use in fact the equivalence classes $f \bigtriangleup$ and $f \bigtriangleup$ of $f$. For example,

$$f \bigtriangleup = \{ g, \exists K \in [0, 255], f = g \bigtriangleup K \text{ or } g = f \bigtriangleup K \}$$

This explains that the result of $d_{\text{As}}(f \bigtriangleup, g \bigtriangleup)$ is insensitive to lighting variations simulated by the LIP addition of a constant.
**ASPLUND’s Metrics**

**Perspective:** We have seen that *LIP operators are particularly efficient to enhance low-light images*. Nevertheless, such an enhancement obviously increase together the signal and the noise. On another hand, *Asplund’s metrics are strongly insensitive to lighting variations and particularly to underlighting, but they remain sensitive to noise*. In such conditions, it appears that a step of noise filtering can improve the potential of Asplund’s metrics. Some preliminary works show that *noise filtering obtained by means of “Deep Learning” approaches* performs well.

Thus we can expect to dispose of powerful tools to perform images acquired under variable lighting.
References


Image Processing under Variable Lighting: LIP Model and Asplund’s Metrics


